OCT 23 1922

VOLUME LVI NUMBER 2

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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SEPTEMBER 1922

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THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

THE CAMBRIDGE UNIVERSITY PRESS, London
THE MARUZEN-KABUSHIKI-KAISHA, Tokyo, Osaka, Kyoto, Fukuoka, Sendai
THE MISSION BOOK COMPANY, Shanghai

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The following are authorized to quote the prices indicated:

For the British Empire: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, £1 16*s.* each; single copies, including postage, 4*s.* 6*d.* each.

For Japan and Korea: The Maruzen-Kabushiki-Kaisha, 11 to 16 Nihonbashi Tori Sanchome, Tokyo, Japan. Yearly subscriptions, including postage, Yen 13.65 each; single copies, including postage, Yen 1.68 each.

For China: The Mission Book Company, 13 North Szechuen Road, Shanghai. Yearly subscriptions, $6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, if mailed direct outside of Shanghai, on yearly subscriptions 50 cents, on single copies 5 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Ryerson Laboratory, University of Chicago, Chicago, Illinois.

The cable address is "University, Chicago."

The articles in this Journal are indexed in the *International Index to Periodicals*, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Post-office at Chicago, Ill., under the act of March 3, 1879.

Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 15, 1918.

PRINTED IN THE U.S.A.

THE
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VOLUME LVI SEPTEMBER 1922 NUMBER 2

REVISION OF THE SERIES IN THE SPECTRUM OF STRONTIUM

By F. A. SAUNDERS

ABSTRACT

Series of triplets and singlets in the spectrum of strontium.—These series belong to the neutral atom of Sr. Their remarkable similarity to the corresponding series of Ca, which have previously been critically studied, was of great assistance in working out the structure. The present study is based on measurements of spectrograms, from ultra-violet to infra-red, taken by Randall, King, and the author, of light from a great variety of sources, including sparks, arc in air and in vacuo, tube-resistance furnace and red-hot quartz discharge tube containing very pure Sr vapor. Revised *wave-lengths* are given for most of the 180 lines from 0.22 to 3.06 μ, including some *seventy new lines*. The limits of all the series have been more accurately found, and the various "terms" have been calculated. Details are given, with tables, of four type series of triplets and four of singlets, four combination series in the triplet system, three in the singlet system, and five inter-system combination series. Of the *twenty series* here mentioned, some are very faint and are represented by one term only, and about half are believed to be new. Evidence is also presented of the existence of *new types of singlet series* corresponding to formulae $(1P)-(mX)$, $(1p)-(mX)$, $(1P)-(mY)$, etc. The terms are all large, so that the series cannot be of the sort suggested by Sommerfeld.

New type of series of singlets in the spectrum of calcium.—Three singlet lines are found to correspond accurately to $(1P)-(mX)$, $(1p_1)-(mX)$ and $(1p_2)-(mX)$ where (mX) is 8584.8. This points to the existence of series similar to those last mentioned for Sr.

The spectra of Ba and Ca have recently[1] been critically examined in the attempt, only partially successful, to arrange all of their lines into series. The present article is to give an account of similar results for Sr. As before, attention is here directed to the

[1] *Astrophysical Journal*, **51**, 23, 1920, and **52**, 265, 1920.

spectrum of the normal atom only, excluding those series (pairs), which are due to the atom (Sr^+) in an ionized condition. These will, it is hoped, be the subject of a later communication.

The spectrum, thus restricted, consists of series of triplets and of singlets, and these form two systems. Each system contains at least four types of series, with combination series formed from these; and there are combinations also formed between the two systems. The amazing similarity between the spectra of Ca and of Sr, extending as it does down to the minutest details, makes it easy to deduce the structure of one from that of the other. Rydberg[1] has indeed laid the two down in parallel columns, and the new data admit of like treatment.

The observations upon which the present work is based comprise photographs taken with grating and with prism instruments of a great variety of sources. Of the greatest value in this connection have been some of the admirable oven spectra taken by Dr. A. S. King, and kindly sent to me for study. These were spectra of a mixture of Sr and Ba, and on this account the weaker Sr lines did not appear, but the plates were very useful on account of their high dispersion (up to 1.86 A per mm) and their sharpness. Many close doubles were thus separated. The use of the oven under reduced pressure also made it possible to obtain as sharp fine lines certain Sr lines which are usually (e.g., arc in air) very much broadened. To supplement these plates, I took others, using as source a red-hot quartz tube, well exhausted, and supplied with metallic Sr, through whose vapor a discharge from a small 10,000-volt transformer was passed. The metal I owe to the kindness of Dr. B. L. Glascock,[2] who prepared it with great care, so that it is almost entirely free from impurities, and contains, in particular, almost no Ca. The spectrum of this source is in striking contrast to the ordinary Sr spectrum, through the absence of the usual oxide bands, and the sharpness of all the lines. The oven, moreover, encourages certain lines, of King's temperature Classes I and II, which I find to belong to the singlet system of series, and on this account very many new ones were observed. For work in the

[1] *Wied. Ann.*, **52**, 119, 1894.

[2] See *Journal of the American Chemical Society*, **32**, 1222, 1910.

visible part of the spectrum I am indebted to the Rumford Fund Committee for a grant which enabled me to construct a large Littrow form of glass spectrograph, which gives excellent spectra, on a fairly large scale (10 inches for the visible spectrum) and makes it possible to carry the search for faint lines down into the beginning of the infra-red. I am also indebted to Dr. Meggers, of the Bureau of Standards, for a small quantity of dicyanin for this purpose.

Other sources also have been used, such as the arc in air and in vacuo and the spark under a variety of conditions. Professor H. M. Randall's recent study of the arc spectrum of Sr (unpublished), concerning the progress of which he has kindly kept me informed by letter, has been of great service. The correct arrangement of lines in series is that which leads accurately to combination lines, which frequently lie in the infra-red; and it has been possible to settle many doubtful points only through such tests. Advantage has also been taken of the observations of Hampe,[1] Lorenser,[2] C. F. Meyer,[3] and Meggers.[4]

The results are presented, as before, with the same notation, under the headings of the individual series. All wave-lengths are on the international system. The numeration (values of m) chosen in my tables differs from that used by Fowler in his *Report on Series in Line Spectra* (1922) only in respect to the fundamental series in both singlet and triplet systems. My values of m for these series must be increased by 2 to bring them into agreement with those of Fowler.

THE TRIPLET SYSTEM

Principal series of triplets: $(1s)-(mp)$.—The first member of this series is shared with the sharp series of triplets; its lines are λλ 7070.28, 6878.12, 6791.06. The corresponding terms $(1p)$ are 31026.8, 31421.1, 31608.0. The second triplet was found by Randall, but its place was predicted through the combination series $(1d)-(mp)$, given below. The predicted wave-lengths are

[1] *Zeitschrift. wiss. Phot.*, **13**, 348, 1914.

[2] Tübingen dissertation, 1913.

[3] *Astrophysical Journal*, **45**, 93, 1917.

[4] *Scientific Papers of Bureau of Standards*, No. 312, 1918.

λλ 20259; 20698; 20762;[1] and those observed by Randall λλ 20263; 20705; 20767. The third triplet can be calculated in the same manner and should be at λλ 9595; 9624; 9637. Randall has observed one line at λ 9597. The fourth triplet has not been positively identified; its strongest line should be near λ 7873, but it is probably extremely faint.

Sharp series of triplets: $(1p)-(ms)$.—Table I gives the wave-lengths of the lines of this series, together with the wave-number differences within each triplet, which should remain constant. The "terms" are also given. The last four lines are believed to be new. The series is of medium strength, well developed in arc and in oven spectra.

TABLE I

Sharp Series of Triplets. $(1p)-(ms)$				Limits 31026.8; 31421.1; 31608.0		
λ I.A.	$\Delta\nu_1$	λ I.A.	$\Delta\nu_2$	λ I.A.	Terms	m
7070.28	395.0	6878.12	186.2	6791.06	16886.8	1
4438.03	394.0	4361.74	186.8	4326.47	8500.9	2
3865.46	395.5	3807.24	186.8	3780.35	5163.2	3
3628.33	393.7	3577.20	188.9	3553.19	3473.4	4
3504.04	391.9	3456.56	187.6	3434.28	2498.0	5
3430.16	394.1	3384.39	.....	.....	1882.0	6
3382.10	394.6	3337.65	.....	.....	1467.7	7

Diffuse series of triplets: $(1p)-(md)$.—This is the most obvious series in the spectrum of Sr, and contains some very strong lines. Of the lines given in Table II some fifteen are believed to be new, and most of the lines have been redetermined on plates taken from vacuum sources, where the lines lose their characteristic diffuseness and become accurately measurable. The complexity of the triplets is thus disclosed, and there appears a trace of the same abnormality in the spacing of these complexities as was noted in the corresponding series in Ca, though the observations are here less complete. The "curve of residuals" also takes a form somewhat like that for Ca and given in Figure 1 of the Ca article referred to above.[2]

[1] See footnote on p. 78.

[2] *Astrophysical Journal*, **52**, 265, 1920.

TABLE II

Diffuse Series of Triplets. $(1p)-(md)$				Limits Same as in Sharp Series		
λ I.A.	$\Delta\nu_2$	λ I.A.	$\Delta\nu_2$	λ I.A.	Terms	m
30665	394.7	27355.3	187.1	26023.6	27606.0	1
30109.7	394.3	26914.5	.....	.....	27706.4	.....
29225.0	.....	.....	.....	.....	27766.0	.....
4971.65	394.2	4876.06	186.7	4832.08	10880.5	2
4967.93	394.0	4872.51	.....	.....	10903.3	.....
4962.31	.....	.....	.....	.....	10918.3	.....
4033.19	394.2	3970.05	186.9	3940.81	6222.2	3
4032.39	394.3	3969.27	.....	.....	6234.5	.....
4030.39	.....	.....	.....	.....	6239.4	.....
.....	.....	3653.93	186.9	3629.14	4051.0	4
3706.74	395.0	3653.26	.....	.....	4056.5	.....
3705.98	.....	.....	.....	.....	4061.1	.....
.....	.....	3500.11	186.8	3477.37	2850.6	5
3548.65	394.2	3499.69	.....	.....	2855.1	.....
3548.09	.....	.....	.....	.....	2858.7	.....
.....	.....	.....	.....	3390.67	2116.4	6
3458.47	394.2	3411.94	.....	.....	2120.5	.....
3457.98	.....	.....	.....	.....	2123.7	.....
.....	.....	.....	.....	3336.13	1634.0	7
.....	.....	3356.69	.....	.....	1638.4	.....
3401.22	.....	.....	.....	.....	1641.7	.....
.....	.....	.....	.....	.....	1300.3	8
3363.84	395.0	3319.72	.....	.....	1307.0	.....
3363.04	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	1060.5	9
3336.58	392.7	3293.42	.....	.....	1065.4	.....
3336.13	.....	.....	.....	.....	.....	.....
3316.32	.....	.....	.....	.....	881.5	10
"Covered" by other strong lines			.....	.....	.....	11
3289.74	.....	.....	.....	.....	638.0	12
3280.54	.....	.....	.....	.....	552.8	13

Fundamental series of triplets: $(1d)-(mf)$.—This series was first found by Fowler. It has been remeasured and several close pairs resolved in it. From λ 3852 on, all the lines are new. Its triplets are complex, like those of the corresponding Ba series, but very close.

TABLE III

Fundamental Series of Triplets. $(1d)-(mf)$				Limits 27606.0; 27706.6.1; 27766.0		
λ I.A.	$\Delta\nu_1$	λ I.A.	$\Delta\nu_2$	λ I.A.	Terms	m
4893.12	100.3	4869.19	59.7	4855.08	7170.2	1
4892.69	100.5	4868.74	.....	.....	7172.9	.....
4892.03	.....	.....	.....	.....	7174.6	.....
.....	.....	4319.12	58.7	4308.07	4558.7	2
4337.89	100.6	4319.03	.....	.....	4559.6	.....
4337.70	.....	.....	.....	.....	4560.4	.....
4087.46	99.5	4070.88	59.2	4060.92	3147.6	3
3950.65	100.1	3935.10	59.7	3925.86	2301.1	4
3866.98	99.8	3852.11	60.1	3843.12	1753.5	5
3811.94	99.6	3797.50	59.5	3788.94	1380.5	6
3773.76	99.6	3759.62	59.3	3751.26	1115.3	7
3746.13	99.7	3732.19	60.4	3723.80	919.7	8
3725.24	.....	.....	.....	.....	769.6	9
3708.87	.....	.....	.....	.....	651.2	10

COMBINATION SERIES IN THE TRIPLET SYSTEM

Series $(1d)-(mp)$.—If we subtract the three terms $(2p)$, which are 11952.2, 12056.9, and 12071.5 (or 12098.2 ?[1]) from the three terms $(1d)$ given above, we obtain nine wave-numbers of which six give real lines in a group in the red, first completely observed by Lorenser. This group is the second member of this series, the first being the first of the diffuse series triplets, taken with negative sign i.e., $-(1p)+(1d)$. The third member of this series is composed of a group of six lines at λλ 4729.52; 4713.96; 4707.10; 4703.93;

[1] There is a curious difficulty here. The term 12071.5 corresponds to a line, λ 6369.91, in the group $(1d)-(2p)$, which has the right intensity, but is, by analogy with Ca, too close to the next group line, λ 6363.88. The corresponding $(1s)-(2p)$ line is λ 20767, which appears (Randall) to have the correct intensity. If, however, we were to consider λ 6380.72 as the true line in the $(1d)-(2p)$ group, we should also find a $(1s-2p)$ line at λ 20882 in the right position and of a tolerable intensity; but a new objection arises from the fact that λ 6380.72 appears to belong also to another group of lines, whose series significance is not yet clear, and observations on the Zeeman effect (Paschen; unpublished) indicate that this classification is correct. Hence, since analogy is a less trustworthy guide, I have discarded this second alternative.

4700.75; 4693.89; which yield terms $(3p)$ as follows: 6467.8; 6498.7; 6513.1. By means of these the third triplet of the principal series was calculated. The fourth member of the series may begin with a line at λ 4269.20, but, if so, others of this group are too faint to have been observed.

Series $(2d)-(mf)$.—This series is represented by a triplet in the infra-red at λλ 26947; 26806; 26714, given by the combination $(2d)-(1f)$. The next member does not appear, doubtless on account of faintness.

Series $(2p)-(md)$.—This series is also represented by a triplet in the infra-red at λλ 17446; 17170; 17137, given by the combination $(2p)-(3d)$. The next member is not found.

Series $(2p_1)-(ms)$.—One line, λ 28964, appears to be the combination $(2p_1)-(2s)$. No others have been found. This combination was not found in Ca. The region in which it lies in Ba has not yet been investigated. It is probably real.

THE SINGLET SYSTEM

Principal series of singlets: $(1S)-(mP)$.—This series is the one formerly called $SL1$.[1] Its later lines have been measured, and the limit much more accurately obtained. Table IV gives numerical data of this series. The terms there given are averages obtained from this series and the combination series $(1D)-(mP)$ given below (p. 82).

TABLE IV

SYSTEM OF SINGLET SERIES

	PRINCIPAL $(1S)-(mP)$		SHARP $(1P)-(mS)$		DIFFUSE $(1P)-(mD)$		FUNDAMENTAL $(1D)-(mF)$	
m	λ I.A.	Terms	λ I.A.	Terms	λ I.A.	Terms	λ I.A.	Terms
1....	4607.34	24227.1	−4607.34	45925.6	−(6.45μ)	25776.3	5156.07	6387.0
2....	2931.88	11827.5	11242.3	15334.5	7621.54	11110.0	4678.35	4406.9
3....	2569.50	7019.0	5970.10	7481.6	5543.42	6192.4	4406.05	3086.8
4....	2428.11	4753.5	5165.45	4873.1	4965.54	4093.7	4252.97	2269.9
5....	2354.32	3463.4	4783.93	3329.6	4688.59	2904.7	4158.49	1735.8
6....	2307.39	2598.8	4582.91	2412.8	4527.29	2145.0	4096.14	1370.0
7....	2275.29	1989.2	4463.31	1828.3	.....	.....	4052.47	1106.9
8....	2253.32	1559.8	.....	.....	.....	.....	4021.02	913.9
9....	2237.65	1251.0	.....	.....	.....	.....	3997.16	765.6
10....	2226.38	1022.0	.....	.....	.....	.....	3978.47	648.1

[1] *Astrophysical Journal*, **32**, 153, 1910.

Sharp series of singlets: $(1P)-(mS)$.—There is the same difficulty in discovering this series as arose in the case of Ca, and the arrangement given in Table IV must be regarded as tentative, especially as regards the last three lines. λ 4783.93 appears on one of my plates as a wing on the strong line λ 4784.39, from which it is not clearly resolved; and there is some question whether a faint line at λ 4585.92 should not be put in place of λ 4582.91. The last four lines given in the table are new, and indicate to what an extent the oven brings out the lines of these rather retiring series.

Diffuse series of singlets: $(1P)-(mD)$.—In harmony with the arrangement already adopted in Ba and Ca, the term 25776.3, which occurs as the limit of the two series formerly called $SL2$ and $SL3$, is considered to be $1D$, the first term of the diffuse series. The second line in this series should lie at the edge of the infra-red, and the choice shown in Table IV must be regarded as somewhat uncertain. There are several good lines in this neighborhood, and a selection among them is difficult. Lacking Stark-effect observations to guide us, we must turn to combinations which would result in consequence of our choice. A number of such combinations occur, but in no case is their arrangement precisely similar to the case of Ca. The choice for the second line of the series seems to lie between λ 7621.54 and λ 7673.11, both of which give interesting combinations. As far as the combinations go, λ 7673.11 is more like the Ca line selected for this place (λ 7326.10), but it fits less well into the "curve of residuals" for the series. It is not now possible to make a final decision on this point. The rest of the series, however, seems fairly certain, and the form of the curve of residuals is exactly like the one for the same series in Ca.

Fundamental series of singlets: $(1D)-(mF)$.—This is the series formerly known as $SL3$. Its lines, given in Table IV, are remeasured with much greater precision, and six new ones have been added. Its terms are, as usual with this type, not far from those of the series in hydrogen.

Indications of new types of singlet series.—Both Ca and Sr show a few strong lines which do not fit into the conventional series scheme, as above outlined, and yet which show, through their

Zeeman effect and their temperature classification, so far as known, that they belong to the singlet system. Sommerfeld has suggested, following an indication given earlier by Paschen, that there are more than four types of series; and the new types should approach more and more nearly to the hydrogen type. Apparently their terms can never be large, and must be less, in fact, than those of the fundamental series. Any indications, therefore, of the existence of new types of series whose terms are *not* small enough to harmonize with Sommerfeld's suggestion, would be of considerable interest, as tending to show that these spectra contain greater complexities than the present atomic model suggests. Such terms we may call (mX), (mY), etc., without attempting to specify what sort of series they may belong to.

If $(mX)=8964.5$, I find that $(1P)-(mX)$ leads exactly to λ 6550.25 (calculated $\nu=15262.6$; observed, 15262.4); $(1p_1)-(mX)$ leads to λ 4531.32 (calculated $\nu=22062.3$; observed, 22062.4); $(1p_2)-(mX)$ leads to λ 4451.80 (calculated $\nu=22456.6$; observed, 22456.3); $(1p_3)-mX)$ leads to λ 4414.87 (calculated $\nu=22643.5$; observed, 22644.5, but some doubt as to whether this is a true Sr line). The corresponding term in Ca is 8584.8, and $(1P)-(mX)$, $(1p_1)-(mX)$, $(1p_2)-(mX)$ there lead to observed lines at $\lambda\lambda$ 5857.48; 3935.36; and 3918.90, respectively, with great accuracy. I cannot find a second term for this series.

If $(mY)=11198.2$, then $(1P)-(mY)$ leads to λ 7673.11 (calculated $\nu=13028.9$; observed, same), and $(1p_2)-(mY)$ to λ 5041.85 (calculated $\nu=19828.6$; observed, 19828.5).

If $(mZ)=8765.2$, then $(1P)-(mZ)$ leads to λ 6465.79 (calculated $\nu=15461.9$; observed, 15461.8); $(1p_1)-(mZ)$ leads to λ 4491.01, a faint oven line (calculated $\nu=22261.6$; observed, 22260.5, perhaps an accidental agreement) and $(1p_2)-(mZ)$ leads to λ 4412.62 (calculated $\nu=22655.9$; observed, 22656.0).

Several of the lines involved in these combinations are new and fairly strong in the oven spectra.

All of these terms are large, and it seems unavoidable to conclude that if they belong to new types of series, as one would expect from the existence of the foregoing combinations, such series cannot be of the sort contemplated by Sommerfeld.

COMBINATION SERIES IN THE SINGLET SERIES

Series $(1D)-(mP)$.—This is the series formerly called SL_2. Its lines have been remeasured, and three new ones found. The revised wave-lengths run as follows: λλ 7167.29; 5329.82; 4755.57; 4480.54; 4313.18; 4202.81; 4128.24; 4076.32; 4038.53. The corresponding terms have been given in Table IV on page 79. This is the strongest of the combination series of the singlet system, and is very well brought out in oven spectra.

Series $(1S)-(mD)$.—This is a faint series, whose existence seems certain, in spite of rather defective observations. The first line occurs at λ 4961.50 and was found by Dr. King precisely in its predicted position, on one of his high dispersion plates, and clearly separated from its strong neighbor λ 4962.31 of the diffuse triplet series. The second member should appear at λ 2871.43. A faint, diffuse line appears in the arc in air near λ 2871.00 which is probably the line in question. The third member has not been found, but a faint line at λ 2389.3 may be the fourth member of the series.

Series $(2S)-(mP)$.—One line of this series seems to have been found by Randall, namely $(2S)-(2P)$; calculated λ 28505; observed, 28516. The agreement is not very satisfactory.

INTER-SYSTEM COMBINATION SERIES

Series $(1S)-(mp_2)$.—This series is well known, on account of the importance of its first line in connection with radiation potentials. This line is at λ 6892.60, as was first pointed out by Lorenser. The second was found in oven spectra at precisely the calculated place, λ 2951.71. The third line is too faint to be visible on any of my photographs.

Series $(2S)-(mp_2)$.—The first line of this series may be the line observed by Randall at λ 30482, though the calculated position is 12 A lower.

Series $(1D)-(mp)$.—One triplet of this series occurs, $(1D)-(2p)$ or at least two of its lines, at λλ 7232.24 and 7287.44, reasonably near their calculated positions. The third line $(1D)-(2p_3)$ should coincide with λ 7309.46, which is too strong, relative to the others, to be formed from this combination. Perhaps there are two lines, not resolved, at this place.

Series $(1p)-(mS)$.—Since $(1p)$ has three values, this combination calls for three series, of which only one appears, namely $(1p_2)-(mS)$ and that very imperfectly. The combination $(1p_2)-(3S)$ leads to λ 4175.84, a faint line in oven spectra, with an accuracy of about 0.1 A.

Series $(1p)-(mD)$.—Lines occur at $(1p_2)-(3D)$ and $(1p_2)-(4D)$, $\lambda\lambda$ 3962.61 and 3658.23, both new oven lines, the first rather strong. The agreement with calculation is satisfactory, the differences being less than 0.1 A. No other line of the series could be found. Another line, λ 4025.52, of the same character, seems to be $(1p_1)-(3D)$, and possibly the combination $(1p_1)-(4D)$ also occurs, at λ 3711.90, which is a faint oven line. It seems likely, therefore, that both series $(1p_1)-(mD)$ and $(1p_2)-(mD)$ occur. If $(1p_3)-(mD)$ occurs, it must be much fainter than the others.

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July 1922

FURTHER EVIDENCE ON THE BRIGHTNESS OF THE STARS OF THE NORTH POLAR SEQUENCE[1]

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ABSTRACT

Mount Wilson photographic and photo-visual scales of magnitude.—These scales were based wholly on plates made with the 60-inch reflector and as there was some question as to photographic magnitudes brighter than the tenth, and photo-visual magnitudes fainter than the eighth, *additional data* have recently been secured with the 10-inch Cooke refractor. Exposures both with and without a wire screen which absorbs 3.02 magnitudes were made on 54 plates. Taking the photographic magnitude of each star to be $0^{M}.19\ C$ less for the 10-inch than for the 60-inch, where C is the color-index, and the photo-visual magnitude to be the same for both, the investigation reveals no appreciable error in the photographic scale and shows it to be homogeneous from the fourth to the fourteenth magnitude; and the slight correction of $+0^{M}.05$ indicated for the photo-visual scale near the ninth magnitude is not confirmed by more reliable exposure-ratio results to be reported elsewhere. The original scales are therefore retained.

Revised magnitudes of stars of the Polar Sequence.—The plates also enable corrections to be given to the photographic magnitudes of 45 stars and to the photo-visual magnitudes of 28 stars. These average $\pm 0^{M}.03$ and $\pm 0^{M}.04$, respectively, for the two scales. The larger photographic corrections are in substantial agreement with those recently found at Greenwich by Jones.

The following pages summarize a recent investigation of the brightness of stars near the North Pole, undertaken in connection with the work of the International Commission on Stellar Magnitudes. Certain results of the earlier investigation[2] had not yet been fully confirmed by observations made elsewhere,[3] and, as the original measures had been wholly with the 60-inch reflector, it seemed desirable that additional data be secured with an entirely different instrument.

The question at issue related mainly to the photographic magnitudes of stars brighter than the tenth magnitude. From the tenth to the fifteenth or sixteenth magnitudes there was evidence that the scale had been well determined; but between the sixth

[1] *Contributions from the Mount Wilson Observatory*, No. 234.

[2] *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, **41**, 206, 1915.

[3] Besides short series of measures by Parkhurst and Schwarzschild, King's results, *Harvard Annals*, **76**, No. 10, were in agreement with Mount Wilson. Jones's observations, *Monthly Notices*, **82**, 21, 1921, which are also in excellent agreement, appeared after the present investigation was begun.

and tenth magnitudes there remained an unexplained difference of a quarter of a magnitude between the Harvard and Mount Wilson results.

Visual magnitudes of the stars of the Polar Sequence have been measured at the Harvard Observatory to the thirteenth magnitude. Besides these and the Mount Wilson photo-visual results published simultaneously with the photographic magnitudes in *Mt. Wilson Contribution* No. 97, there is no evidence bearing on the visual or photo-visual scale of the stars in the polar region fainter than the eighth magnitude. Further investigation of this scale was therefore also desirable.

The instrument used for the present investigation was the 10-inch Cooke refractor of 45 inches focal length, which represents a radical change in conditions as compared with the 60-inch reflector. Its relatively small aperture implies an entirely different relation between diffraction pattern and resulting photographic image of a star; its small focal length means minimized influence of atmospheric tremor; its large field results in small corrections for distance error (for a compact group of stars at least); while its large color equation presents an extreme example of the difference in the color-sensibility that may affect the instrumental equipment.

The reduced-intensity exposures were made with a wire screen absorbing about three magnitudes, placed in front of the objective. As usual, equal multiple exposures, arranged in a symmetrical sequence (for example: full aperture, screen, screen, full aperture), were made on each plate, the exposures for different plates ranging from 1 to 32 minutes in order that stars of widely differing brightness might be accurately measured.

The longer exposures for photographic brightness extend several magnitudes below the questionable portion of the scale. This greatly simplified the reductions; for since the photographic scale below the tenth magnitude may be accepted as reliable, the magnitudes in this region could be used to determine the absorption produced by the screen. Moreover, as will appear shortly, a series of photographs covering a wide range in exposure time affords a simple means of testing the homogeneity of the scale without actually calculating the magnitudes of individual stars. This

calculation is eventually necessary, but it is convenient to see in advance if a divergence in scale is to be expected.

The photographs were taken and measured by Humason. For much of the calculation connected with their reduction we are greatly indebted to Miss Joyner. For the determination of the photographic scale 43 plates were available. The investigation of the photo-visual scale, which depends on only 11 plates, could be much abbreviated, since the results are supplemented by independent evidence obtained by combining the adopted photographic magnitudes with values of the color-indices derived by the method of exposure ratios. To restrict the labor as much as possible, only stars of the Polar Sequence were measured. Corrections for distance from the center of the plate were neglected, since the distances themselves are small. Only a few of the brighter stars are more than a degree or two from the Pole, and these were brought to the center of the plate by shifting the telescope before making the screen exposures.

The measurement of each photograph yielded two series of scale readings, one for full-aperture and one for screen exposure. These were first used to determine the screen constant and to test for homogeneity the scales of *Contribution* No. 97.

Since these scales refer to the color-system of the reflector, the individual magnitudes must be reduced to the system of the 10-inch refractor before they can be used with the latter instrument or compared with results obtained with it. The differences in the color-sensibility of different telescopes (together with the photographic plate) are of the same order as those in the visual color-perception of different observers. In careful work these instrumental peculiarities cannot be neglected, but in practice the data necessary for reduction to a normal system of color are usually lacking. It is not difficult to determine the relative color-equation of two instruments, but the second essential—the color of the star whose brightness is to be determined—is generally unknown, and the comparison of results obtained with different instruments then means little unless the relative color equations are small. In the present case, however, the colors of the stars are known.

The chief instrumental source of the color-equation lies in the violet and ultra-violet absorption occurring in the optical system. Since light of these colors plays little part in forming a photo-visual image, the relative color-equations for photo-visual magnitudes are much smaller than those for photographic, as long as the same brand of plate and the same filter are used. In the present instance we adopt provisionally for the reduction to the system of the 10-inch refractor[1]

$$m_{10} = m_{60} - 0.19\,C \text{ (Photographic)} \qquad (1)$$

$$m_{10} = m_{60} \text{ (Photo-visual)} \qquad (2)$$

where m_{60} and C are magnitudes and color-indices given in *Contribution* No. 97. The resulting values of m_{10} are in the second column of Tables III and VI.

These magnitudes were plotted against the two series of scale readings (full-aperture and screen images, respectively) derived from each plate. In the ideal case of perfectly accurate magnitudes and no errors of observation, the plotted points would define two parallel curves, whose separations, measured parallel to the magnitude (ordinate) axis, would be equal to the value of the screen constant. Actually the behavior of the curves indicates the degree of homogeneity in the given scale. In the present case the photographic scale below the tenth magnitude is known to be substantially correct, and we should expect the curves in this region to be sensibly parallel. The screen constant is determined from this portion of the data.

Detailed results for the differences in the ordinates of the curves are given for individual plates in Tables I and II, omitting only those whose screen exposures show but one or two stars each. The first column indicates the ordinate (magnitude) of the full-aperture curve from which the distance between the curves was measured. The values of the separation for every half-magnitude are shown in the body of the tables in the form of deviations from an even three magnitudes. Thus the first value in Table I for plate 517, exposure 32 *m*, indicates a separation of 3.10 mag., measured

[1] Seares and Hubble, *Mt. Wilson Contr.*, No. 187; *Astrophysical Journal*, **52**, 8, 1920.

TABLE I

Screen Constant, Photographic Data

Mag.	32ms		16ms		8ms					4ms					2ms		
	517	525	508	509	479	493	494	495	496	478	489	490	491	492	477	485	486
9.0	....	....	....	....	+ 8	....	....	....	....	0	+2	−30	0	−15	0	−7	+8
9.5	....	....	+10	−5	+10	+23	+11	+20	+10	− 2	0	−18	0	− 8	−7	−4	+5
10.0	....	....	+ 6	−5	+10	+20	+10	+13	+ 7	+ 5	+2	− 8	−3	− 3	−4	−3	+2
10.5	....	....	+ 2	−5	+ 9	+16	+11	+ 8	+ 4	+12	+4	+ 3	0	0	−5	0	−3
11.0	....	+ 8	− 1	−6	+10	+14	+14	+ 5	+ 1	+18	+4	+ 4	0	+ 3	−4	+5	−2
11.5	+10	+ 1	− 5	−5	+10	+17	+20	+ 4	+ 6	+18	+2	+ 3	+6	+ 8	−1	0	−2
12.0	+ 4	− 7	− 5	−5	+10	+20	+20	+10	+10	+20	+5	0	+9	+13	+2	−4	−4
12.5	0	−16	−10	−1	+ 6	+17	+20	+14	+10	+16	+6	− 4	+4	+14	+8	−5	−1
13.0	− 2	−17	− 9	−2	0	+10	+21	+16	+10	+ 4	+4	− 4	+3	+ 9	....	−4	−1
13.5	− 6	−16	− 9	−4	0	+10	+ 4	+ 8	0	....	+3	+ 2	0	+ 2	....	....	....
14.0	+ 5	− 8	− 4	−7	− 2	+10	+11	0	− 2	....	....	....	....	....	....	....	....
14.5	+15	− 4	0	−5	....	....	....	....	....	....	....	....	....	....	....	....	....
15.0	+20	− 2	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
Mean	+ 6	− 7	− 2	−5	+ 7	+16	+14	+10	+ 6	+10	+3	− 5	+2	+ 2	−1	−2	0
S.C.	− 4	+ 9	+ 4	+7	− 5	−14	−12	− 8	− 4	− 8	−1	+ 7	0	0	+3	+4	+2

Mag.	2ms		1ms										No.	Mean(1)	Mean(2)
	487	488	476	481	482	483	484	510	511	512	513	514			
9.0	+ 4	0	0	+15	−18	−5	−12	+14	+22	− 2	+5	+15	21	3.00	3.00
9.5	+ 3	0	+6	+11	− 9	−2	− 7	+14	+10	− 3	+7	+13	27	3.03	3.02
10.0	+ 4	+ 1	+6	+ 7	0	0	− 2	+ 9	− 2	+ 2	+7	+18	27	3.04	3.02
10.5	+ 5	0	+6	+ 4	+ 1	+3	+ 2	+ 2	−11	− 2	+9	+20	27	3.04	3.02
11.0	+ 8	− 2	+4	0	0	+2	+ 2	+ 2	−11	− 9	+4	+22	28	3.03	3.02
11.5	+10	− 1	−4	− 5	0	0	+ 2	+ 5	−13	−20	+2	+20	29	3.03	3.02
12.0	+13	+ 4	....	....	− 2	0	+ 2	+ 4	−12	−23	−4	+22	27	3.04	3.03
12.5	+15	+10	....	....	....	+6	+ 2	....	....	....	....	....	21	3.05	3.04
13.0	+20	+15	....	....	....	....	....	....	....	....	....	....	18	3.04	3.02
13.5	....	....	....	....	....	....	....	....	....	....	....	....	13	3.00	2.98
14.0	....	....	....	....	....	....	....	....	....	....	....	....	9	3.00	2.97
14.5	....	....	....	....	....	....	....	....	....	....	....	....	4	3.02	3.05
15.0	....	....	....	....	....	....	....	....	....	....	....	....	2	3.09	3.12
Mean	+ 9	+ 3	+3	+ 5	− 4	0	− 1	+ 7	− 2	− 8	+4	+19	253	3.03	3.02
S.C.	− 7	− 1	−1	− 3	+ 6	+2	+ 3	− 5	+ 4	+10	−2	−17	....	....	....

TABLE II

Screen Constant, Photo-visual Data

Mag.	32ms			16ms				8ms				No.	Mean (1)	Mean (2)
	526	527	543	539	540	541	542	535	536	537	538			
9.0	+5	−11	− 6	+6	− 5	0	+4	− 4	−14	+1	+ 2	11	2.98	2.98
9.5	+6	−15	− 6	+4	0	+2	+6	− 2	− 4	0	+ 4	11	3.00	2.99
10.0	+5	−14	− 9	+2	+ 7	+9	+4	+ 5	+ 7	0	+ 6	11	3.02	3.02
10.5	+5	−10	−14	+3	+10	+8	+4	+15	+13	+5	+10	11	3.04	3.04
11.0	+4	− 5	−14	−1	+14	0	+8	+23	+20	+6	+ 7	11	3.06	3.05
11.5	+2	− 1	−10	−6	+ 8	−6	+8	....	....	....	....	7	2.99	3.00
12.0	+2	0	0	....	....	....	....	....	....	....	....	3	3.01	3.06
12.5	+4	+ 2	+ 7	....	....	....	....	....	....	....	....	3	3.04	3.09
Mean	+4	− 7	− 6	+1	+ 6	+2	+6	+ 7	+ 4	+2	+ 6	68	3.02	3.02
S.C.	−2	+ 9	+ 8	+1	− 4	0	−4	− 5	− 2	0	− 4	....	....	....

from the point on the full-aperture curve whose ordinate is magnitude 11.5. For plate 508, on the other hand, the separation for the same ordinate is 2.95 mag.

Restricting our attention for the moment to the photographic scale (Table I), we note that the two values just given depend upon stars whose mean magnitudes are 11.5 and 8.5, respectively, those of the latter value having been reduced to an apparent brightness of 11.5 by the use of the screen. Were the magnitudes of these stars as given in *Contribution* No. 97 certainly dependable, the numbers 3.10 and 2.95 could be accepted at once as approximations for the screen constant. But since we assume the scale to be known only for stars fainter than magnitude 10, it follows that only values of the separation of the curves for ordinates 13.0 and fainter are available for the calculation of the constant.

We note, however, that the means of the quantities in each line of Table I, which are given in the last column but one under the heading "Mean (1)," are very nearly constant. There is no evidence of a progressive change, and this shows at once that the photographic scale of *Contribution* No. 97 is sensibly homogeneous, for, no matter what portion of this scale between the sixth and fifteenth magnitudes is used, we find appreciably the same value of the screen constant. Assuming the scale to be correct from the tenth magnitude downward, that between the sixth and tenth magnitudes must also be substantially correct. In the mean, the photographic data in Table I give the following result:

$$\left.\begin{array}{l}\text{Stars between mags. 6 and 10, } \Delta m = 3.03 \text{ (207 values)} \\ \text{Stars between mags. 10 and 15, } \Delta m = 3.02 \text{ (46 values)}\end{array}\right\} \quad (3)$$

The value 3.02 mag. is therefore adopted for the screen constant. There is so little progressive change in the values of Mean (1) that the treatment of the photographic data reduces itself to a simple combination, arranged to diminish as much as possible the accidental errors in the individual magnitudes.

The agreement in the values of Mean (1) for the photo-visual results shown in Table II is nearly as satisfactory. The irregularities are a little larger and there seems to be a small progressive

change, but the number of plates included in each mean is much smaller than in Table I. From all the data in Table II,

$$\text{Stars between mags. 6 and } 12.5,\ \Delta m = 3.02 \text{ (68 values)} \qquad (4)$$

which is the same as the adopted constant found from the photographic data. We conclude, therefore, that the photo-visual scale is very nearly, if not quite, homogeneous; moreover, (4) shows that in the mean it is consistent with the photographic scale, and hence should be substantially correct. We shall examine the small progressive change in the values of Mean (1), Table II, in a later paragraph.

The individual values of Δm in both Tables I and II show the influence of appreciable systematic errors which are peculiar to a plate. The means for the separate plates shown at the bottom of the tables range from 2.92 to 3.19 (plates 512 and 514) and 2.93 to 3.07 (plates 527 and 535), respectively, with an average deviation of ± 0.05 mag. This is only to be expected from observations which depend upon multiple exposures. Fluctuations in atmospheric conditions, the absorption of water vapor by the photographic film, and a variety of other factors combine to produce an effective value of the screen constant for each plate which differs more or less from the true value. The disturbance can be rendered accidental by increasing the number of plates, as in the present investigation, where its influence upon the adopted constant is very small.

The effect upon the magnitudes of individual stars will be more serious, however, unless special precautions are taken, for some of the stars appear on a few plates only. The difficulty can be avoided by reducing each plate with its effective constant; but we must first see whether this will modify the scale. This may be determined by applying to each value of Δm in each column the corresponding systematic correction in the last lines of Tables I and II, the corrections being the difference between 3.02 and the mean Δm for the plate in question. The horizontal means of the results thus found are given under the heading "Mean (2)," and represent what would have appeared under Mean (1), had the effective Δm for each plate been equal to 3.02. The revised values are sensibly the same as

those of Mean (1), which shows that the scale will not be altered by reducing the individual plates with their effective constants. By so proceeding the errors in the individual magnitudes are appreciably diminished.

Since Table I gives no evidence of appreciable error in the photographic scale of *Contribution* No. 97, the determination of the magnitudes is a simple matter. It is only necessary to read from the full-aperture curves referred to above, with scale reading as argument, the magnitude of each star, and subtract from those interpolated with readings of images obtained with the screen, the values of the effective constant. Since the effective constant cannot be determined when only one or two screen images appear, several photographs of bright stars were reduced with the mean constant 3.02 mag.

Each of the magnitudes thus found yields a correction to the corresponding magnitude given in *Contribution* No. 97. The results are summarized in Table III, which shows the mean correction derived from each group of plates, classified according to exposure time. The second column of the table gives the magnitude from *Contribution* No. 97, reduced to the color-system of the 10-inch refractor. The headings of the six following columns indicate the exposure and the number of plates in each group. The corrections in the body of the table are given separately for full-aperture (left-hand series) and screen exposures. The number of values included in each mean correction is usually the same as the number of plates in the group, except in the case of the 1-minute plates. For these the number varies from 1 to 24, with an average of 9. Individual corrections in excess of 0.3 mag., 4 in a total of 954, were rejected. The weighted means from all the data and the total numbers of values appear in the last two columns.

The corrections for stars 1, 4, 3*s*, and 1*r*, and for the faintest stars in the list are of relatively low weight. The largest among the corrections of high weight is −0.08 mag. The average deviation of a single magnitude is ±0.070 mag. The simple mean of the corrections based on five or more determinations is ±0.028 mag. The probable reality of the larger corrections is indicated by the agreement shown in Table IV with similar corrections found by

Jones,[1] the quantities in parentheses being the approximate relative weights within the respective series. There are only two cases of

TABLE III

CORRECTIONS TO PHOTOGRAPHIC MAGNITUDES

Star	Pg. m_{pg}	Exposure and Number of Plates						No.	Mean Corrn.
		$32^m, 2$	$16^m, 2$	$8^m, 5$	$4^m, 5$	$2^m, 5$	$1^m, 24$		
1.....	4.39	.....	.....	.....	.....	.....	+ 7	4	+ 7
4.....	5.90	.....	.....	.....	.....	.....	+14	4	+14
25.....	6.43	.....	.....	+ 9	− 4	+ 4	− 2	25	+ 1
5.....	6.45	.....	.....	+ 6	−11	− 5	−10	20	− 5
35.....	6.58	.....	.....	.....	.....	.....	− 6	5	− 6
37.....	6.32	.....	.....	.....	.....	.....	+ 9	6	+ 9
6.....	7.10	.....	+ 4	− 5	+ 1	+ 2	+14, 0	26	+ 2
7.....	7.43	.....	− 8	− 5	+ 3	− 4	+ 8, + 4	36	+ 2
27.....	7.64	.....	+ 5	+ 3	+ 1	− 5	− 9, + 6	31	− 1
8.....	8.28	− 5	+10	+ 2	+10	0	+ 2, 0	34	+ 3
37.....	8.69	0	+ 4	+ 5	+ 7	− 4	− 6, − 1	38	0
9.....	8.87	+10	+ 2	+ 3	+10, +14	+ 5, +10	+ 3, − 2	54	+ 5
10.....	9.11	+ 1	+ 8	0, +11	− 7, +10	− 4, + 6	− 2, − 4	57	+ 1
47.....	9.04	− 6	−11	+11, − 5	+ 1	+ 3, + 2	+ 1, − 3	35	− 1
11.....	9.69	0	+ 9, − 6	+ 1, +10	− 3, − 1	+ 3, − 8	− 4	49	− 1
12.....	10.03	−14	+ 1, −10	− 5, 0	− 8, − 8	− 3	− 2	46	− 4
57.....	9.85	−20	+ 8	0	+ 7, + 2	− 1, − 6	+ 3	30	+ 1
43.....	10.17	−16	−10	+ 1, + 9	+ 5, 0	+ 3	+ 2	38	+ 1
13.....	10.50	−19	− 1, − 6	− 2, −10	− 1	+ 2	+ 7	35	− 1
67.....	10.22	− 4	+ 2	+ 4, + 2	− 2, + 1	+ 2	− 1	38	0
14.....	10.90	− 4, − 8	−10, + 8	− 4, −22	− 3	−10	−11	37	− 8
77.....	10.74	+12, + 7	+ 4, − 3	+ 7, − 3	− 1	+ 1	− 1	35	+ 1
53.....	10.89	− 3	.....	.....	.....	+12	−12	14	− 4
15.....	11.16	− 3, + 8	+ 6, + 2	+ 7	+ 8	+ 5	+ 8	33	+ 6
65.....	11.26	−2	− 9	+ 1	+ 4	− 4	0	24	− 1
87.....	11.25	+ 2, + 6	0, − 4	+ 5	+ 6	− 4	0	29	+ 2
16.....	11.55	− 9, 0	+ 1	0	− 2	− 3	0	22	− 2
17.....	11.76	0, +19	+ 5	+ 1	+ 2	+ 6	+ 1	29	+ 3
18.....	12.20	+11, +11	+ 9	0	− 4	+ 2	− 7	23	+ 1
19.....	12.61	0	+ 2	+ 1	− 1	− 2	.....	18	0
20.....	12.92	+ 2	+ 4	− 4	0	− 9	.....	16	− 3
21.....	13.17	− 3	− 2	− 1	+ 4	.....	.....	11	0
117.....	13.00	.....	− 4	+10	+11	.....	.....	7	+ 6
22.....	13.32	+18	− 4	+ 7	.....	.....	.....	8	+ 7
23.....	13.50	+ 6	+12	+ 6	.....	.....	.....	8	+ 7
127.....	13.58	+ 2	.....	.....	.....	.....	.....	2	+ 2
24.....	13.82	+ 7	− 2	− 1	.....	.....	.....	7	0
25.....	13.99	+ 4	+ 5	.....	.....	.....	.....	4	+ 4
85.....	14.36	−10	.....	.....	.....	.....	.....	2	−10
26.....	14.47	−13	− 3	.....	.....	.....	.....	3	−10
95.....	14.58	−16	.....	.....	.....	.....	.....	2	−16
27.....	14.80	− 6	.....	.....	.....	.....	.....	2	− 6
105.....	15.14	+16	.....	.....	.....	.....	.....	2	+16
115.....	15.13	− 2	.....	.....	.....	.....	.....	2	− 2

an actual difference in algebraic sign, which shows a close accordance between the recent Greenwich investigation and that reported here.

[1] *Loc. cit.*

For the seven red stars of high weight, 2*r* to 8*r* inclusive, there are two zero differences, four of 0.01 mag., and one of 0.02 mag. This confirms the reality of the large color equation of the 10-inch refractor for photographic magnitudes, and shows that the adopted coefficient in equation (1) is sensibly correct.

TABLE IV

COMPARISON WITH CORRECTIONS FOUND BY JONES

Star	Table III	Jones	Star	Table III	Jones
1*r*	+9(6)	+11(11)	5	−5(20)	+1(26)
14	−8(37)	−10(109)	12	−4(46)	0(113)
15	+6(33)	+ 5(96)	5*s*	−4(14)	−6(108)
11*r*	+6(7)	+ 7(8)	8	+3(34)	+3(39)
3*s*	−6(5)	+ 1(9)	17	+3(29)	+2(56)
9	+5(54)	+10(33)	.....	.....	.....

The corrections in the last column of Table III may be combined with the magnitudes in the second column to form weighted mean values referred to the system of the 10-inch refractor, or with the photographic magnitudes of *Contribution* No. 97, to obtain a revised system for the 60-inch reflector. Other data are available, however; and the final results, based on all the material which has been accumulated, are given in the following *Contribution* No. 235.

Turning now to the photo-visual magnitudes, consider first the small progressive change in the screen constant indicated by the last two columns of Table II. This suggests that the photo-visual scale requires a slight readjustment. The smoothed values of the constant are given in the second column of Table V, and it is readily seen that each of these would become equal to the adopted mean, were the original scale modified by the quantities shown in the third column of the table. Thus, since the first value of Δm depends upon the difference in brightness of stars whose mean magnitudes are 6.0 and 9.0, respectively, corrections of 0.00 and +0.05 at these two points of the scale would increase the observed value from 2.97 to 3.02. Again, the last value of Δm depends upon stars of mean magnitude 9.5 and 12.5. Corrections of +0.04 and +0.01, respectively, give the required reduction of 0.03 from 3.05 to the adopted mean. The maximum correction is +0.05 mag. in the vicinity of magnitude 9.

These results, however, scarcely justify a change in the original scale, for the divergence in question arises almost wholly from two of the eleven plates, Nos. 535 and 536, both of which show large differences in gradation between the full-aperture and the screen exposures (see Table II). There is, moreover, additional evidence. The colors of the stars near the Pole have been extensively observed by the method of exposure ratios, and since we now have reason to believe that the photographic scale for the brighter stars is reliable, it is clear that the photo-visual scale may be independently established by combining photographic magnitudes with color-indices derived from the exposure ratios. This material is discussed in *Contribution* No. 235, but the results are summarized in the last four columns of Table V, the significant quantities appearing under the heading "Corrn. (2)."

TABLE V

SCALE CORRECTIONS

Pv. Mag.	Δm	Scale Corrn. (1)	Pv. Mag.	Scale Corrn. (2)	No. Stars	Rel. Weight
6.0	.....	0	3.0	−3	2	3
6.5	.....	+2	5.4	+3	4	4
7.0	.....	+3	6.5	0	5	3
7.5	.....	+4	8.0	−7	5	4
8.0	.....	+4	9.5	−9	6	10
8.5	.....	+5	10.4	−1	5	22
9.0	2.97	+5	11.1	−1	5	14
9.5	3.00	+4	12.4	+3	14	38
10.0	3.02	+3	13.5	+3	22	22
10.5	3.03	+3	14.3	+1	28	17
11.0	3.04	+2	15.4	−1	40	10
11.5	3.05	+2	16.2	+3	31	7
12.0	3.05	+2	16.9	−2	49	6
12.5	3.05	+1	.....	.....	.....	.....

With the exception of the values for 8.0 and 9.5 it will be noted that these corrections are small and in general accidental in character. Further, the two large corrections are opposite in sign to the values of Corrn. (1). In view of these results, which are of good weight, it appears that Corrn. (1) is of the order of the uncertainty affecting its determination, and may be disregarded. Nor does Corrn. (2) itself afford any certain basis for a change in the opposite direction. The exposure-ratio observations in

these two groups are not well distributed, and most of the difference can be traced to three or four stars. A small systematic effect is certainly present, but as between the two series of magnitudes and the exposure ratios, its source cannot be located. The photo-visual scale of *Contribution* No. 97, as well as the photographic, is therefore adopted unchanged.

The problem of photo-visual magnitudes, provisionally at least, is therefore also one of diminishing the accidental errors of the original values in *Contribution* No. 97. Proceeding as for the photographic data, we find the results given in Table VI.

TABLE VI

PHOTO-VISUAL MAGNITUDES OF POLAR SEQUENCE STARS

Star	Pv. Mag. *Cont.* 97	Sums of Deviations +,−	No.	Corrn.	Star	Pv. Mag. *Cont.* 97	Sums of Deviations +,−	No.	Corrn.
2S	6.33	5,76	11	−0M06	4S	9.83	35, 27	11	+0M01
2F	6.35	8,73	11	− 6	7F	9.90	0,114	11	− 10
5	6.47	0,38	11	− 3	13	10.37	29, 17	11	+ 1
6	7.05	32,18	11	+ 1	8F	10.44	55, 11	11	+ 4
7	7.52	80, 7	10	+ 7	14	10.54	50, 0	11	+ 5
3F	7.56	36,12	11	+ 2	6S	10.73	0, 17	2	− 8
8	8.13	24,34	7	− 1	15	10.89	17, 30	11	− 1
4F	8.26	23,12	6	+ 2	16	11.23	24, 7	7	+ 2
5F	8.65	0,31	3	− 10	17	11.28	39, 0	7	+ 6
9	8.81	54,11	7	+ 6	18	11.89	11, 2	3	+ 3
10	9.07	19,46	12	− 2	11F	12.06	8, 0	2	+ 4
6F	9.21	81,22	13	+ 5	19	12.28	0, 39	3	− 13
11	9.53	69, 8	13	+ 5	21	12.48	16, 0	2	+ 8
12	9.80	7,82	11	−0.07	20	12.51	0, 30	2	−0.15

The magnitudes in the second column are taken directly from *Contribution* No. 97. The third column contains the sums of the positive and negative deviations for the 11 plates; the fourth column, the total number of values; and the fifth, the mean corrections to the magnitudes in the second column. Thus the first correction is $(+5-76)/11=-6$.

Neglecting three rejected corrections which exceed 0.3 mag., we find for the mean of the 231 remaining deviations ± 0.065. The weighted algebraic mean of the final corrections for the red stars is less than 0.01 mag. This shows that for photo-visual magnitudes the color-systems of the 10-inch refractor and the reflector are identical, and justifies the assumption expressed by equation (2).

As in the case of Table III, the corrections may be combined with the original magnitudes to form revised values. Those finally

adopted, like the photographic results, also depend on data drawn from other sources and are given in *Contribution* No. 235.

In résumé, the investigation with the 10-inch refractor reveals no appreciable error in the photographic scale of *Contribution* No. 97 between the fourth and fourteenth magnitudes. The photovisual scale has been examined between magnitudes 6.3 and 12.5. A maximum correction of +0.05 mag. near the ninth magnitude is suggested, but independent data of higher weight do not confirm this result. The original scale is therefore adopted. The corrections to the magnitudes of *Contribution* No. 97 for accidental error (5 or more values) average ±0.03 and ±0.04 mag., respectively, for the two scales. The larger corrections for the photographic scale are in substantial agreement with those recently found at Greenwich by Jones.

MOUNT WILSON OBSERVATORY
February 1922

REVISED MAGNITUDES FOR STARS NEAR THE NORTH POLE[1]

By FREDERICK H. SEARES

ABSTRACT

Revised magnitudes and color-indices for stars near the North Pole.—Numerous comparisons with the polar standards have incidentally given extensive material for determining the errors affecting the standards themselves. The results for 55 stars (Table I) lead to revised magnitudes which are averaged with previously reported results obtained with the 60-inch reflector and the 10-inch Cooke refractor to give the adopted values listed in Table VIII. This table also gives the corresponding color indices from magnitudes and from exposure-ratios for these and 37 other polar stars, all based on the color system of the 60-inch reflector and the accessory equipment.

Exposure-ratio method of determining color-indices has been previously described. A discussion of the results of 177 exposures including stars of photo-visual magnitude ranging from 2 to 17.3, indicates that the average deviation of a single determination is ±0.12 magnitude, about half of which is due to the mean systematic error of the plate, and that in general the *probable error* is about 30 per cent less than in the case of a color-index derived by combining a photographic and a photo-visual magnitude. *Normal points for the calibration curve*, color-index as a function of log E, determined by observations on 216 stars, are given in Table IV and the adopted values in Table V.

Differential error between photo-visual and photographic scales of magnitude.—Although the exposure-ratio color scale is calibrated by measurements on stars of known color index, systematic errors in the two magnitude scales upon which the color scale depends were eliminated by using stars with a wide range of magnitude. Hence the exposure-ratio scale is independent of the magnitude scales and may be used to test the differential error between them. The data (Table VII) show agreement to within $\pm 0^{M}.03$ from photo-visual magnitude 3 to 17, except for a few stars near magnitude 9 for which the mean difference is $0^{M}.09$.

The following pages bring together the results of several investigations bearing upon the magnitudes of stars near the North Pole. The basic determination of both photographic and photo-visual scales is that of *Mt. Wilson Contribution* No. 97,[2] which depends entirely upon observations with the 60-inch reflector. These results refer to the color-system of that instrument as used with the photographic plates[3] and a certain yellow filter habitually employed in photometric work at Mount Wilson.

To diminish the effect of accidental errors, the individual magnitudes of about fifty of the stars in *Contribution* No. 97 have been combined with a large amount of material obtained incidentally in connection with an entirely different investigation.

[1] *Contributions from the Mount Wilson Observatory*, No. 235.

[2] *Astrophysical Journal*, **41**, 206, 1915.

[3] Seed's Gilt Edge, 27's and 30's, and Cramer's Instantaneous Isochromatic.

The revised magnitudes are on the scale and color-system of *Contribution* No. 97.

At various times during the last six years numerous measures of the colors of stars near the Pole have been made by the method of exposure ratios. The method must be calibrated upon stars of known color-index, but this can be accomplished in such a manner that systematic errors in the magnitude scales upon which the colors themselves depend do not enter into the calibration curve. Colors found from exposure ratios are thus essentially independent of the scale, and may therefore be used to test for differential error between photographic and photo-visual scales, since in the mean there should be no difference between the colors found by the two methods. A discussion of these data shows excellent agreement to the seventeenth photo-visual magnitude, except for a few stars near the ninth magnitude for which the mean difference is 0.09 mag.

Observations made at Greenwich, Harvard, and Potsdam confirm the substantial accuracy of the photographic scale of *Contribution* No. 97 to the sixteenth photographic magnitude.

A recent investigation made at Mount Wilson[1] with the 10-inch Cooke refractor affords further evidence on both the scales. The photographic scale is again confirmed. The photo-visual results are less reliable, but, considered in connection with the measures of colors, show that the scale of *Contribution* No. 97 must also be close to the truth.

These several investigations are combined to form a system of magnitudes embodying all the data bearing on standard polar stars thus far collected at Mount Wilson. The final results, which are on the color-system of the reflector and the accessory equipment mentioned above, represent the contribution of this observatory to the report of the International Commission on Stellar Magnitudes for 1922.

1. REVISION TO DIMINISH ACCIDENTAL ERROR

In determining the magnitudes of faint stars in the Selected Areas[2] it has been necessary to make a large number of comparisons,

[1] Seares and Humason, *Mt. Wilson Contr.*, No. 234. *Astrophysical Journal*, **56**, 84, 1922.

[2] See recent "Annual Reports of the Director of the Mount Wilson Observatory," *Year Book of the Carnegie Institution of Washington.*

both photographic and photo-visual, with the polar standards of *Contribution* No. 97. Since the comparisons were intended only to establish the zero point of the magnitudes in the Selected Areas, and not the scale, the exposures were short, 2^m and 3^m, respectively. They afford, nevertheless, extensive material for the reduction of the accidental errors affecting the polar standards themselves.

We are here concerned only with exposures on the Pole of which there are two on each plate. As a part of the reduction, the scale readings for these had been plotted against the adopted standards in the usual manner. The deviations of individual stars from the smooth curves defined by the plotted points represent the combined influence of the accidental errors in the standards and the errors affecting individual plates. In the mean the latter are accidental, whereas the former enter systematically into the observed deviations from the curves and should be brought to light by forming the algebraic mean of a large number of such deviations.

Table I summarizes the results of this revision. The second, third, and fourth columns contain the mean systematic deviation for each star, obtained as just explained, and the number of values upon which each mean depends. The deviations are the corrections required by the magnitudes in *Contribution* No. 97 to bring them into agreement with the results of the polar comparisons. The fifth and sixth columns give the revised magnitudes. These are means of the values in *Contribution* No. 97 and those derived from the polar comparison plates, formed with weights equal to the respective numbers involved. The last two columns give the revised color-indices and the total numbers of individual observations for each star.

A few corrections are large and indicate cases of more or less serious discordance. Most of these are the result of low weight in one determination or the other.[1] For example, the photographic magnitude of star 439 given in *Contribution* No. 97 is clearly in error by a large amount and is rejected outright. Again, star 358 shows a difference of 0.47 mag. in the photo-visual magnitudes. Here both determinations are uncertain, and the adopted mean is

[1] Note that the numbers of values included in the means in *Contribution* No. 97 are equal to the differences in corresponding quantities in the fourth and last columns of Table I.

TABLE I

MAGNITUDES AND COLORS DERIVED WITH 60-INCH REFLECTOR

STAR	N.P.C.—*Contr.* 97			REVISED MAGS. AND C.I.			
	Pg.	Pv.	No.	Pg.	Pv.	C.I.	No.
14	+ 5	+ 3	37, 9	11.00	10.55	0.45	94,27
16	− 8	− 4	21,13	11.59	11.21	0.38	64,29
19	+ 5	− 5	57,54	12.71	12.24	0.47	135,66
20	− 7	+ 2	69,60	12.99	12.53	0.46	155,72
22	+15	+ 6	27,25	13.47	12.84	0.63	72,36
23	− 2	−12	69,58	13.59	13.00	0.59	149,70
24	+ 1	− 3	44,34	13.93	13.31	0.62	115,46
25	+ 5	− 2	50,44	14.10	13.58	0.52	128,56
26	− 4	− 4	66,44	14.62	13.69	0.93	140,56
27	−10	−12	62,12	14.86	14.25	0.61	123,19
28	0	+ 4	28, 1	15.27	14.54	0.73	68, 8
31	0	.....	17,..	16.41	15.62	0.79	62, 6
5S	+ 5	0	36, 3	11.11	10.06	1.05	96,31
7S	+10	− 8	44,45	12.65	12.04	0.61	114,57
8S	+10	− 3	29,10	14.53	13.77	0.76	79,22
9S	− 1	−14	68,41	14.74	13.74	1.00	134,53
10S	− 1	+25	22, 1	15.29	14.52	0.77	67, 8
11S	− 2	+ 1	52, 8	15.30	14.35	0.95	115,18
12S	+ 2	− 9	39, 2	15.34	14.67	0.67	100, 9
13S	− 5	.....	22,..	15.52	14.54	0.98	65, 3
14S	+ 3	−15	26, 1	16.00	15.05	0.95	73, 8
15S	− 3	.....	5,..	16.57	15.71	0.86	46, 5
16S	+ 4	.....	1,..	16.86	15.50	1.36	37, 4
11I	+ 1	+ 1	43,41	13.22	12.07	1.15	122,53
12I	− 8	+ 1	69,59	13.81	12.47	1.34	151,71
125	−30	.....	1,..	15.02	14.37	0.65	3, 5
127	− 4	.....	2,..	14.34	13.31	1.03	8,11
130	−21	.....	1,..	15.19	14.21	0.98	3, 9
134	−17	.....	1,..	14.41	13.54	0.87	7,10
151	− 2	+18	3, 9	13.72	12.66	1.06	32,20
152	+12	− 3	3, 1	14.58	13.76	0.82	24,13
158	0	.....	1,..	15.10	14.57	0.53	17, 5
176	+14	−13	5, 4	14.35	13.78	0.57	41,16
226	− 8	−21	25, 4	14.90	14.15	0.75	71,11
256	− 4	− 4	28, 8	14.66	13.84	0.82	78,19
277	+ 6	−15	49, 4	15.33	14.56	0.77	111,11
311	+ 4	− 5	42, 6	15.02	14.34	0.68	95,13
319	+ 9	+ 7	14,25	14.46	13.61	0.85	54,28
329	+10	+19	14,27	14.35	13.51	0.84	52,30
334	− 1	+ 3	51,14	15.20	14.44	0.76	121,21
338	+ 8	+ 6	5,10	14.73	13.84	0.89	33,13
341	− 6	+12	8,15	15.15	14.02	1.13	34,18
345	− 1	+ 5	36,35	13.79	12.05	1.74	94,46
350	+ 3	+14	26,11	11.02	10.71	0.31	72,23
358	+ 9	+47	9, 2	15.24	14.34	0.90	35, 4

TABLE I—*Continued*

Star	N.P.C.—*Contr.* 97			Revised Mags. and C.I.			
	Pg.	Pv.	No.	Pg.	Pv.	C.I.	No.
362.....	−12	− 5	67,61	13.08	12.27	0.81	153,73
378.....	− 2	+ 8	64,54	14.13	12.81	1.32	118,59
389.....	+ 8	+ 8	48,43	13.61	12.90	0.71	91,51
391.....	− 2	− 2	46,32	14.62	13.83	0.79	84,38
401.....	− 8	+10	25,33	14.50	13.70	0.80	61,36
403.....	− 4	+ 9	27,34	14.49	13.83	0.66	68,37
424.....	+ 2	+15	20,26	14.52	13.77	0.75	49,29
426.....	+ 7	+ 6	12,14	13.85	13.38	0.47	40,17
427.....	− 1	− 2	23,41	13.27	12.05	1.22	56,46
439.....	−51	.....	21,..	14.78	13.49	1.29	21, 3

of little value. Occasionally, a difference is large although numerous observations enter into each of the values combined to form the revision. Star 22 is an example, with a difference of 0.15 mag. The most plausible explanation is a small error in the distance correction. The star is at the edge of the useful field; the exposures for the polar comparison plates were always the same, while those for the original investigation were varied. Errors in distance correction tend to neutralize each other in the latter case because the photographic image varies in size from plate to plate, whereas there is no compensation in magnitudes derived from the comparison photographs. On this account the value from the polar comparisons has been given half weight. In general, however, the differences are as satisfactory as can be expected from results derived with a reflector of focal ratio.

It should be noted that the revision concerns only the accidental errors affecting the original magnitudes. The reliability of the scales is a separate question, discussed partly in *Contribution* No. 234 and partly in the following sections of this paper.

2. COLORS OF POLAR STARS FROM EXPOSURE RATIOS

The method of exposure ratios for measuring a star's color has been described briefly in previous communications.[1] The term

[1] *Mt. Wilson Comm.*, Nos. 33, 38, 59; *Proc. Nat. Acad. Sci.*, **2**, 521, 1916; **3**, 29, 1917; **5**, 232, 1919. The second of these contains provisional results of the investigation discussed in this section.

"exposure ratio" is used to designate the logarithmic ratio of exposure times (symbol, log E) which produce blue and yellow images of the same size. This quantity can obviously be used as a measure of color.

Experience shows that for a given star the values of log E from plates of the same emulsion are nearly constant. The systematic differences in the means from groups of 5 or 6 plates each are usually negligible. Observations are now generally arranged in such a way as to check, if not actually determine, the zero point for each plate, but in the beginning a direct control, beyond that of accordance with other plates, was not ordinarily applied, the errors being minimized by multiplying the number of plates. In the case of bright stars, which must be observed individually, it is now customary to photograph several objects on the same plate, which greatly facilitates the reduction to a homogeneous system. In so far as possible, the same emulsion was used in all the earlier measures. When a change became necessary, the constant for reduction to a uniform system was carefully determined.

To translate values of log E into color-indices, which is an important step in giving them physical meaning, stars of known color-index must be measured. Inasmuch as the stars in the polar region are the basis of the Mount Wilson photometric system, they have been extensively observed for this purpose. The resulting data give not only the desired relation between color-index and log E, but also an independent determination of the colors of the individual stars. The exposures for the yellow images range from 8^s to 64^m, and results are now available for an interval of about 16 magnitudes, the lower limit being approximately 17 of the photo-visual scale. The details of interest in the present connection are as follows:

Series a.—About 150 plates; yellow exposures mostly 16^s and 32^s, on stars brighter than the eleventh magnitude, observed with images of standard size. A few plates of longer exposure extend the series to the thirteenth photo-visual magnitude.

Series b.—Nine plates with yellow exposures of 128^s. The faintest stars observed are of about the fourteenth magnitude.

Series c.—Three plates with yellow exposures of 8^m, three of 10^m, and two of 30^m. The limiting magnitudes range from 15.5 to 16.9.

Series d.—Nine plates with yellow exposures ranging from 8^s to 5^m. Only 17 stars between 9.0 and 12.5 were included, however, because the longer exposures were made with reduced apertures. This group of plates, along with several others included in the various series, was made under unusual conditions to test various details of the method which are of no interest in the present connection.

Series e.—Four plates, each including 10 to 15 stars brighter than the tenth magnitude, observed individually with yellow exposures of 8^s to 32^s and a standard size of image. In addition, observations with yellow exposures of from 32^s to 128^s were made at the beginning and end, to include stars between magnitudes 10.0 and 12.5. The results supplement and strengthen those of series *a*.

Series f.—Two plates with yellow exposures of 64^m each. The limiting magnitude is 17.3 photo-visual.

Lack of space prevents a detailed presentation of the reduction of these data, which has involved much time and several revisions as the results have accumulated. The discussion is therefore limited to a general statement of procedure, with extracts from the reduction sheets showing the accordance attained, supplemented by a collection of results, made particularly from the standpoint of their bearing on the magnitude scales.

The earlier observations were restricted to stars brighter than the eleventh photo-visual magnitude, because of the ease with which certain systematic errors could thereby be eliminated. It is well known that a photographic image produced by yellow light usually grows at a different rate from one produced by blue light; the value of log E therefore depends to some extent upon the size of the images from which it is derived. Differences in photographic gradation for light of different wave-lengths seem to depend upon a number of factors. In practice these are hard to control, and the most satisfactory results are obtained when the observations are arranged in such a way as to eliminate the disturbance or to permit its determination for each plate.

In the case of the preliminary measures this was accomplished by reducing all the stars to a standard intensity with the aid of

screens and diaphragms and by using an essentially constant series of exposures throughout. Those for the yellow images were 16 and 32 seconds, although a few photographs with exposures ranging from 1 to 4 minutes were subsequently added. The latter gave results for bright stars already observed with short exposures, as well as for fainter stars, the overlap being sufficient to determine the zero-point and the correction to be applied for gradation. In practice the two corrections were combined. The differences in log E for stars within the overlapping region, when arranged in order of photo-visual magnitude, define a curve which could be extrapolated for an interval of one or two magnitudes without danger of serious error.

The discussion of the data of series *a* carried the results to the thirteenth magnitude. To extend them to still fainter stars, the procedure just described was applied in succession to the plates of series *b*, *c*, and *f* in the order of increasing exposure time. Series *d* and *e*, which control and strengthen the results of series *a*, were inserted and reduced in the order indicated by the series notation. The adopted values of C_e, the color-index derived from log E, and the number of determinations upon which each depends are given in the eighteenth and nineteenth columns of Table VIII, although results for a few stars, mostly of low weight, not included in the final collection of data, appear in Table VI.

The plates of series *f* show many faint stars for which individual results are not printed, although the means for groups of these stars appear in some of the statistical comparisons (see Table IV, for example).

The question as to how much the values of log E for faint stars have been affected by errors incident upon the extrapolation of the correction curve for gradation is important. The results for a single plate are systematically in error by small amounts at least; but for a series of plates these errors become accidental. An examination of the residuals for series *c*, in the interval 14.1 to 15.7, shows that the effect on the mean must be small. Although some of the plates of series *b* contribute results for two or three stars falling within the interval mentioned, the gradation curves for series *c* are in fact extrapolations from 13.7. The systematic deviations of individual

plates from the mean, expressed in hundredths of a magnitude, are

Systematic deviation	− 9	− 1	− 8	+ 6	+ 8	+ 4	−5	+10
No. of stars.....	13	13	20	11	19	16	5	4

The average is ±0.064 mag., whence the probable value of the systematic error in the mean color-indices derived from log E for stars in this interval is of the order of ±0.02 mag. This could have been depressed to a still lower value by adding more plates, but the precision is sufficient for the present purpose.

As a further indication of accordance, it may be added that from the same data the average deviation for a single value of log E, including systematic errors, corresponds to ±0.118 mag. For stars in the interval between the tenth and thirteenth magnitudes, on the other hand, the agreement is as shown in Table II, which compares the mean results for each star as derived from series *a* to *e*. The table gives the deviations in log E in units of the third decimal and the number of values upon which each difference depends. There is here little evidence of residual systematic error; and since 0.010 in log E is equivalent, on the average, to 0.028 mag., the accidental deviations are also satisfactory. The combined effect of both types of error is indicated by the average of the 43 deviations based on 7, 8, or 9 observations each, whose value is ±0.013 in log E, or ±0.036 mag.

Finally, an examination of some of the bright stars in series *a*, which were observed individually with a standard size of image, throws light upon the fluctuations in the relative blue and yellow sensitiveness of different plates. Since for these stars no attempt was made to determine the zero-point correction for individual plates, the average deviations include the full effect of this error, combined with the accidental error of estimating the value of log E. Further, the mean log E usually depends upon two or even three different emulsions. The results have been referred to a standard emulsion, but the reduction constants are affected by an uncertainty which also enters into the average deviations.

The results for 15 of these stars which have been extensively observed are shown in Table III. The second and third columns

give the number of plates and of emulsions used. In every case the observations extend over a period of more than a year, and many of the later measures were made with the original emulsion with which the observations were begun. There are small systematic

TABLE II

DEVIATIONS IN LOG E FOR DIFFERENT SERIES

Unit = 0.001 in log E; No. of Values in Parentheses

STAR	PV. MAG.	SERIES				
		a	*b*	*c*	*d*	*e*
7r	9.90	− 5 (31)	−11 (9)	+25 (3)	+ 6 (9)	+30 (8)
5s	10.06	0 (35)	+ 5 (9)	0 (4)	− 2 (9)	+15 (7)
13	10.37	− 1 (14)	+18 (3)	.....	.....	.....
8r	10.44	− 1 (15)	+10 (6)	+ 7 (5)	.....	.....
14	10.55	+ 6 (24)	+23 (9)	− 5 (7)	+16 (9)	−59 (6)
350	10.71	− 8 (24)	− 5 (7)	+11 (3)	+16 (9)	+26 (6)
6s	10.73	+10 (12)	+21 (8)	− 6 (6)	−23 (7)	.....
15	10.89	+ 5 (15)	+23 (6)	−12 (6)	− 9 (9)	.....
16	11.21	−32 (3)	− 1 (8)	+ 6 (8)	− 4 (9)	.....
17	11.28	−28 (6)	− 3 (9)	+53 (6)	.....	.....
18	11.89	−16 (4)	− 1 (9)	+25 (8)	.....	.....
345	12.05	− 1 (11)	−10 (7)	−36 (2)	+52 (3)	+ 24 (4)
11r	12.07	− 3 (15)	+17 (9)	− 6 (8)	+17 (5)	− 9 (4)
427	12.05	−63 (1)	+ 6 (8)	+19 (8)	− 5 (5)	.....
7s	12.04	+ 5 (18)	+11 (9)	+14 (7)	−30 (6)	+ 2 (6)
172	12.24	.....	+42 (1)	+28 (2)	.....	.....
19	12.24	+20 (9)	− 9 (9)	− 9 (8)	+13 (5)	+ 7 (4)
362	12.27	+ 7 (19)	0 (9)	−27 (8)	+ 6 (5)	+43 (4)
12r	12.47	.....	+22 (9)	−10 (8)	.....	.....
21	12.48	.....	− 4 (6)	+13 (8)	.....	.....
20	12.53	+21 (18)	−23 (8)	−17 (8)	+42 (5)	−37 (4)
151	12.66	.....	+37 (3)	− 9 (8)	.....	.....
378	12.81	+19 (16)	− 8 (6)	−24 (8)	.....	.....
22	12.85	.....	0 (5)	+ 8 (8)	.....	.....
389	12.90	.....	− 8 (7)	+18 (8)	.....	.....
23	13.00	+17 (13)	+ 8 (6)	−25 (8)	.....	.....

differences affecting groups of plates of nearly the same date, but little evidence of actual changes in the color-properties of an emulsion with increasing age. The average deviations for a single plate in the fourth column show considerable fluctuation; their mean is ±0.106 mag. From other evidence it seems likely that this error is to be about equally divided between the neglected plate constants and the uncertainty in estimating the value of log E.

As mentioned above, it is now customary to determine the plate constant by comparison with standard fields, either directly

or indirectly, which increases the precision appreciably. The results just discussed show, however, that even when the variations in color-sensitiveness from plate to plate are neglected, the exposure ratios lead to values of the color which are at least comparable in accuracy with those found by other methods of direct color-measurement. The errors given above are of the same order, for example, as those affecting individual magnitudes in the general run of photometric work. The uncertainty of a color-index derived by combining a photographic and a photo-visual magnitude

TABLE III

AVERAGE DEVIATION IN COLOR INDEX. BRIGHT STARS

Star	No. Obs.	No. Em.	A.D.	Star	No. Obs.	No. Em.	A.D.
			mag.				mag.
10.....	23	2	±0.135	7s.....	16	2	±0.124
13.....	14	1	.092	11.....	13	1	.119
14.....	24	2	.084	3r.....	9	2	.093
15.....	15	2	.097	7r.....	31	2	.099
20.....	18	2	.105	8r.....	15	1	.072
18.....	39	3	.097	10r.....	8	1	.138
5s.....	35	2	.111	11r.....	15	2	±0.155
6s.....	12	1	±0.062				

should therefore be greater than that of a value from log E in the ratio of about 1.4 to 1. This is a fair statement of the advantage of the method of exposure ratios from the standpoint of precision.

The calibration curve for the transformation of log E into color-index has been revised several times. The first approximation, which differs little from the adopted relation, was obtained from series *a* by first arranging the mean values of log E for each star in the order of increasing color-index as given in *Contribution* No. 97 and then forming the means of log E and of color-index for groups of stars within narrow limits of color. Accidental irregularities were eliminated by plotting the results and drawing a smooth curve. This curve represents the relation by means of which values of log E may be expressed in the customary form as color-indices.

Since most of the groups include stars scattered over a wide range in brightness, the errors in the photographic and photo-visual scales, which of course appear in the color-indices, are almost

completely neutralized. In the present case, in which the scale errors are known to be small, their effect upon the mean relation between color-index and log E is negligible. Values of the color calculated from log E should therefore be free from errors in the scales, and when compared with the original color-indices arranged in order of increasing magnitude must bring to light any differential error in the scales themselves.

TABLE IV

NORMAL POINTS FOR RELATION BETWEEN COLOR-INDEX AND LOG E

C_m	log E	v	Pv. Interval	No. Stars
mag.		mag.		
0.04	9.040	+0.03	5.3– 8.8	8
0.26	.109	− 1	5.6–10.6	8
0.43	.186	+ 2	2.1–12.5	8
0.58	.229	− 1	11.3–15.2	10
0.67	.268	0	10.7–16.9	15
0.71	.319*	+ 10	15.5–16.8	15
0.81	.299	− 5	12.5–15.6	22
0.88	.367*	+ 7	15.5–17.0	15
0.99	.389	+ 2	8.3–16.3	18
0.99	.365*	− 4	14.8–17.0	15
1.06	.412*	+ 2	15.2–17.0	15
1.15	.410*	− 7	15.5–17.0	15
1.19	.444	− 1	9.2–16.0	17
1.23	.464*	+ 1	15.6–17.3	15
1.34	.485*	− 3	15.1–17.3	15
1.59	9.546	−0.08	5.1–12.0	5
Sum				216

* Supplementary faint stars from series *f*.

To obtain the last approximation for the reduction curve, the method just described was applied to the data from all the series as summarized in Tables VI and VIII, including the slight revisions of color-index given in Table I. The results are shown in Tables IV and V. The former gives the normal points and their representation by the adopted curve in Table V. Table IV also gives seven supplementary points based upon 105 faint stars between photovisual magnitudes 14.8 and 17.3, which appear only on the two plates of long exposure in series *f*. Although of much lower weight, these points show substantially the same change in log E with color-index as the brighter stars. The average deviations of the two series of normal points in Table IV from the adopted curve are

±0.026 and ±0.049 mag., respectively. The average for the points in the first approximation, referred to the *final* curve, is ±0.028 mag. This approximation, it will be recalled, was based exclusively upon series *a*, which includes only stars brighter than the thirteenth magnitude. The large residual of −0.08 for the last point in Table IV arises mainly from the star 345, magnitude 12.05, which, in view of the numerous measures, is seriously discordant. (See Table VIII, last column.) No explanation suggests itself, except possibly systematic error in the estimates caused by the proximity of the brighter object, No. 350, magnitude 10.71.

TABLE V

COLOR-INDEX AND LOGARITHM OF EXPOSURE RATIO

C.I.	log *E*	Diff. for 0.01 in log *E*	C.I.	log *E*	Diff. for 0.01 in log *E*
mag.		mag.	mag.		mag.
−0.2.....	8.940	0.026	0.8.....	9.315	0.028
0.0.....	9.015	.027	1.0.....	9.385	.030
+0.2.....	9.090	.027	1.2.....	9.450	.031
0.4.....	9.165	.027	1.4.....	9.513	.033
0.6.....	9.240	.027	1.6.....	9.573	.034
0.8.....	9.315	0.028	1.8.....	9.632	0.035

The absence of blue stars from the fainter objects is noteworthy, and confirmatory of earlier results. Among the supplementary stars from series *f* there is none with a color-index less than 0.6 mag.

From the ordinates of the adopted relation between color-index and log *E* given in Table V, it appears that the curve is nearly linear. The zero-point for log *E* is of course arbitrary. For comparison with the provisional relation between log *E* and spectral type given in Figure 1, *Mt. Wilson Communications*, No. 59,[1] a correction of −0.07 must be applied to the values of log *E* in Table V. This comparison leads to a result of importance, namely, the relation of the color-indices of the Mount Wilson photometric system to spectral type. Provisional values of the equivalents are listed in Table XII, *Contribution* No. 226. Detailed results and revised equivalents will be given in a later paper.

The data of Table V were used to calculate the color-indices for individual stars whose values, C_e, appear in Tables VI and

[1] *Proc. Nat. Acad. Sci.*, **5**, 232, 1919.

VIII. Subtracted from the values of C_m found by comparing photographic and photo-visual magnitudes, these lead to the differences in the last column of the tables. These differences should be judged in connection with the number of separate determinations of the photographic and photo-visual magnitudes (sixteenth column, Table VIII) which enter into the observed

TABLE VI

OBSERVATIONS OF COLOR

(Stars not included in Table VIII, mostly of low weight)

No. H.A. 48	*Contr.* 97			EXPOSURE RATIOS		$C_m - C_e$
	Pg.	C_m	No.	C_e	No.	
126.....	15.58	0.93	2, 4	1.08	3	−15
139.....	15.30	1.34	2, 7	1.20	4	+14
142.....	14.60	0.89	2,11	0.72	5	+17
143.....	16.35	0.84	7, 2	0.68	7	+16
146.....	14.52	0.96	2,10	0.62	5	+34
154.....	14.95	0.59	8, 9	0.48	5	+11
159.....	15.20	0.70	16, 4	0.79	3	− 9
163.....	15.13	1.20	1, 3	0.82	3	+38
164.....	16.02	0.99	18, 5	0.86	3	+13
172.....	13.85	1.61	2, 1	1.38	4	+23
202.....	16.12	0.71	27, 4	0.78	3	− 7
205.....	15.97	0.89	11, 4	0.78	3	+11
214.....	15.39	0.73	9, 7	0.48	3	+25
215.....	15.47	1.78	10, 3	1.34	3	+44
218.....	15.47	1.36	17, 8	1.38	10	− 2
258.....	16.07	0.85	6, 4	0.64	3	+21
283.....	14.87	1.37	28, 3	1.24	4	+13
295.....	15.57	0.64	32, 5	0.66	8	− 2
301.....	16.23	0.66	26, 2	0.74	3	− 8
336.....	14.92	0.95	26, 5	0.77	5	+18
342.....	14.86	0.77	26, 3	0.64	5	+13
368.....	15.78	1.30	2, 3	1.21	3	+ 9

color-indices, and of C_e (nineteenth column). Well-observed stars usually show a satisfactory agreement, although there are some unexpectedly large values. The case of No. 345 (12.05) has already been mentioned. No. 427 (also 12.05) should also show better agreement than it does. A similar difference for No. 172 (12.24), on the other hand, is not at all surprising, and this is also true of the four differences approximating 0.4 mag., all but one in Table VI. These latter results might have been rejected at once because of uncertainty, but they have been retained to show that

the values already published in *Contribution* No. 97 are open to suspicion. Superficially, one would expect a smaller difference for star 2 (5.28); its case is exceptional in that the values of log E for series a and e are discordant by about 0.4 mag., owing probably to an unfavorable combination of neglected plate constants. The persistent negative sign shown by the Polar Sequence stars near the fifteenth magnitude disappears when the differences for the *H.A.* 48 stars of the same brightness, in the latter part of Table VIII,

TABLE VII

COMPARISON OF COLOR-INDICES FROM MAGNITUDES AND FROM EXPOSURE RATIOS

Pv. Mag.	Series a			Series a–f		
	$C_m - C_e$	No. Stars	No. log E	$C_m - C_e$	No. Stars	No. log E
	mag.			mag.		
3.0	+0.02	2	42	−0.03	2	46
5.4	+ 3	4	21	+ 3	4	36
6.5	0	5	14	0	5	34
8.0	− 5	5	25	− 7	5	39
9.5	− 8	6	65	− 9	6	103
10.4	− 1	5	112	− 1	5	217
11.1	+ 1	6	51	− 1	5	143
12.4	+0.04	8	109	+ 3	14	379
13.5	.....	.....	.....	+ 3	22	216
14.3	.....	.....	.....	+ 1	28	167
15.4	.....	.....	.....	− 1	40	105
16.2	.....	.....	.....	+ 3	31	68
16.9	.....	.....	.....	−0.02	49	59
Sums...	.....	41	439	.....	216	1,612

are also taken into account. The systematic difference near the ninth magnitude is discussed in the following section.

3. CORRECTIONS TO PHOTO-VISUAL SCALE

It should be repeated that the two methods of deriving color-indices are independent, and that the values obtained from log E are essentially free from differential errors in the magnitude scales, as may be seen from the limits of brightness for the stars included in the normal points of Table IV (fourth column). The values of the differences $C_m - C_e$, when combined into means for groups of stars within narrow limits of magnitude, should therefore indicate the relative error in the scales. Since the photographic scale may

TABLE VIII

Mean Magnitudes and Color-Indices

Star	Photographic						Photo-visual						Adopted Magnitude			Color-Index			
	Contr. 97		10-Inch		N.P.C.		*Contr.* 97		10-Inch		N.P.C.		Pg.	Pv.	No.	C_m	C_e	No.	$C_m - C_e$
	Mag.	No.	Diff.	No.	Diff.	No.	Mag.	No.	Diff.	No.	Diff.	No.							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
1S.....	2.54	40	.....	.....	.....	.....	2.08	23	.....	.....	.....	.....	2.54	2.08	40,23	+0.46	+0.51	39	− 5
1.....	4.39	20	+ 7	4	.....	.....	4.37	9	.....	.....	.....	.....	4.40	4.37	24, 9	0.03	−0.01	7	+ 4
2.....	5.30	15	.....	.....	.....	.....	5.28	11	.....	.....	.....	.....	5.30	5.28	15,11	0.02	−0.18	7	+20
3.....	5.83	22	.....	.....	.....	.....	5.56	11	.....	.....	.....	.....	5.83	5.56	22,11	0.27	+0.14	6	+13
4.....	5.91	23	+14	4	.....	.....	5.84	9	.....	.....	.....	.....	5.93	5.84	27, 9	+0.09	0.21	6	−12
5.....	6.45	25	− 5	20	.....	.....	6.47	10	− 3	11	.....	.....	6.43	6.45	45,21	−0.02	0.02	8	− 4
2S.....	6.45	33	+ 1	25	.....	.....	6.33	10	− 6	11	.....	.....	6.45	6.30	58,21	+0.15	0.18	7	− 3
1r.....	6.61	27	+ 9	6	.....	.....	5.09	10	.....	.....	.....	.....	6.63	5.09	33,10	1.54	1.54	17	0
3S.....	6.64	22	− 6	5	.....	.....	6.35	10	.....	.....	.....	.....	6.63	6.35	27,10	0.28	0.23	4	+ 5
6.....	7.11	16	+ 2	26	.....	.....	7.05	10	+ 1	11	.....	.....	7.12	7.06	42,21	+0.06	+0.05	7	+ 1
7.....	7.41	26	+ 2	36	.....	.....	7.52	12	+ 7	10	.....	.....	7.42	7.55	62,22	−0.13	−0.16	7	+ 3
2r.....	7.94	22	− 1	31	.....	.....	6.35	12	− 6	11	.....	.....	7.93	6.32	53,23	+1.61	+1.55	8	+ 6
8.....	8.32	36	+ 3	34	.....	.....	8.13	14	− 1	7	.....	.....	8.33	8.13	70,21	0.20	0.31	8	−11
9.....	8.88	26	+ 5	54	.....	.....	8.81	12	+ 6	7	.....	.....	8.91	8.83	80,19	0.08	0.24	4	−16
3r.....	8.96	19	0	38	.....	.....	7.56	11	+ 2	11	.....	.....	8.96	7.57	57,22	1.39	1.43	13	− 4
10.....	9.12	21	+ 1	57	.....	.....	9.07	12	− 2	12	.....	.....	9.13	9.06	78,24	0.07	0.19	24	−12
4r.....	9.22	20	− 1	35	.....	.....	8.26	9	+ 2	6	.....	.....	9.21	8.27	55,15	0.94	1.10	7	−16
11.....	9.73	28	− 1	49	.....	.....	9.53	10	+ 5	13	.....	.....	9.72	9.56	77,23	0.16	0.26	3	−10
12.....	10.09	25	− 4	46	.....	.....	9.80	12	− 7	11	.....	.....	10.06	9.77	71,23	0.29	0.31	5	− 2
5r.....	10.13	24	+ 1	30	.....	.....	8.65	12	−10	3	.....	.....	10.14	8.63	54,15	1.51	.....	.....	.....
4S.....	10.25	19	+ 1	38	.....	.....	9.83	11	+ 1	11	.....	.....	10.26	9.83	57,22	0.43	0.52	2	− 9
6r.....	10.46	26	0	38	.....	.....	9.21	12	+ 5	13	.....	.....	10.46	9.24	64,25	1.22	1.24	7	− 2
13.....	10.53	19	− 1	35	.....	.....	10.37	16	+ 1	11	.....	.....	10.52	10.37	54,27	0.15	0.29	17	−14
7r.....	10.94	34	+ 1	35	.....	.....	9.90	28	−10	11	.....	.....	10.95	9.87	69,39	1.08	1.13	62	− 5
14.....	10.98	57	− 8	37	+ 5	37	10.54	18	+ 5	11	+ 3	9	10.97	10.56	131,38	0.41	0.48	57	− 7

5s.....	11.09	60	− 4	14	+ 5	36	10.06	28	.....	.....	0	3	11.10	10.06	110,31	+1.04	+1.10	66	− 6
15.....	11.22	30	+ 6	33	.....	.....	10.89	10	− 1	11	.....	.....	11.25	10.88	63,21	0.37	0.37	37	0
6s.....	11.38	9	− 1	24	.....	.....	10.73	13	− 8	2	.....	.....	11.37	10.72	33,15	0.65	0.71	34	− 6
8r.....	11.44	32	+ 2	29	.....	.....	10.44	17	+ 4	11	.....	.....	11.45	10.46	61,28	0.99	0.97	26	+ 2
16.....	11.62	43	− 2	22	− 8	21	11.23	16	+ 2	7	− 4	13	11.60	11.22	86,36	0.38	0.35	30	+ 3
17.....	11.87	43	+ 3	29	.....	.....	11.28	13	+ 6	7	.....	.....	11.88	11.30	72,20	0.58	0.50	21	+ 8
18.....	12.27	49	+ 1	23	.....	.....	11.89	14	+ 3	3	.....	.....	12.27	11.90	72,17	0.37	0.40	21	− 3
10r.....	.....	.....	.....	.....	.....	.....	12.03	5	.....	.....	.....	.....	.....	12.03	5	.....	.....	.....	.....
7s.....	12.61	70	.....	.....	+10	44	12.10	12	.....	.....	− 8	45	12.65	12.04	114,57	0.61	0.56	48	+ 5
19.....	12.69	78	0	18	+ 5	57	12.28	12	−13	3	− 5	54	12.71	12.24	153,69	0.47	0.44	37	+ 3
20.....	13.02	86	− 2	16	− 7	69	12.51	12	−15	2	+ 2	60	12.99	12.52	171,74	0.47	0.54	45	− 7
11r.....	13.22	79	+ 6	7	+ 1	43	12.06	12	+ 4	2	+ 1	41	13.23	12.07	129,55	1.16	1.10	43	+ 6
21.....	13.33	37	0	11	.....	.....	12.48	12	+ 8	2	.....	.....	13.33	12.49	48,14	0.84	0.69	16	+15
22.....	13.44	45	+ 7	8	+15	27	12.81	11	.....	.....	+ 6	25	13.48	12.84	80,36	0.64	0.58	15	+ 6
23.....	13.60	80	+ 7	8	− 2	69	13.10	12	.....	.....	−12	58	13.59	13.00	157,70	0.59	0.53	29	+ 6
12r.....	13.84	82	+ 2	2	− 8	69	12.46	12	.....	.....	+ 1	59	13.80	12.47	153,71	1.33	1.30	19	+ 3
24.....	13.93	71	0	7	+ 1	44	13.33	12	.....	.....	− 3	34	13.93	13.31	122,46	0.62	0.64	13	− 2
25.....	14.08	78	+ 4	4	+ 5	50	13.60	12	.....	.....	− 2	44	14.10	13.58	132,56	0.52	0.62	14	−10
8s.....	14.49	50	−10	2	+10	29	13.78	12	.....	.....	− 3	10	14.52	13.77	81,22	0.75	0.85	12	−10
26.....	14.64	74	−10	3	− 4	66	13.72	12	.....	.....	− 4	44	14.62	13.69	143,56	0.93	1.06	13	−13
9s.....	14.75	66	−16	2	− 1	68	13.85	12	.....	.....	−14	41	14.74	13.74	136,53	1.00	1.08	12	− 8
27.....	14.91	61	− 6	2	−10	62	14.33	7	.....	.....	−12	12	14.86	14.25	125,19	0.61	0.69	10	− 8
28.....	15.27	40	.....	.....	0	28	14.54	7	.....	.....	+ 4	1	15.27	14.54	68, 8	0.73	0.92	5	−19
10s.....	15.29	45	+16	2	− 1	22	14.49	7	.....	.....	+25	1	15.29	14.52	69, 8	0.77	0.92	7	−13
11s.....	15.31	63	− 2	2	− 2	52	14.35	10	.....	.....	+ 1	8	15.30	14.35	117,18	0.95	0.97	10	− 2
12s.....	15.33	61	.....	.....	+ 2	39	14.69	7	.....	.....	− 9	2	15.34	14.67	100, 9	0.67	0.76	7	− 9
13s.....	15.54	43	.....	.....	− 5	22	14.54	3	.....	.....	.....	.....	15.52	14.54	65, 3	0.98	1.10	8	−12
29.....	15.82	40	.....	.....	.....	.....	15.21	6	.....	.....	.....	.....	15.82	15.21	40, 6	0.61	0.75	5	−14
14s.....	15.99	47	.....	.....	+ 3	26	15.07	7	.....	.....	−15	1	16.00	15.05	73, 8	0.95	1.21	7	−26
30.....	16.18	34	.....	.....	.....	.....	15.44	5	.....	.....	.....	.....	16.18	15.44	34, 5	0.74	0.74	5	0

TABLE VIII—*Continued*

Star	Photographic						Photo-visual						Adopted Magnitude			Color-Index			
	Contr. 97		10-Inch		N.P.C.		*Contr.* 97		10-Inch		N.P.C.		Pg.	Pv.	No.	C_m	C_e	No.	$C_m - C_e$
	Mag.	No.	Diff.	No.	Diff.	No.	Mag.	No.	Diff.	No.	Diff.	No.							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
31	16.41	45	.....	.....	0	17	15.62	6	.....	.....	.....	.....	16.41	15.62	62, 6	+0.79	+0.78	5	+ 1
158	16.57	41	.....	.....	− 3	5	15.71	5	.....	.....	.....	.....	16.57	15.71	46, 5	0.86	0.74	4	+12
32	16.76	33	.....	.....	.....	.....	15.58	3	.....	.....	.....	.....	16.76	15.58	33, 3	1.18	1.44	4	−26
168	16.86	36	.....	.....	+ 4	1	15.50	4	.....	.....	.....	.....	16.86	15.50	37, 4	1.36	1.45	4	− 9
33	17.06	32	.....	.....	.....	.....	15.97	3	.....	.....	.....	.....	17.06	15.97	32, 3	1.09	1.11	3	− 2
34	17.24	24	.....	.....	.....	.....	16.29	2	.....	.....	.....	.....	17.24	16.29	24, 2	0.95	1.10	3	−15
35	17.63	20	.....	.....	.....	.....	16.94	2	.....	.....	.....	.....	17.63	16.94	20, 2	0.69	0.80	3	−11
125	15.12	2	.....	.....	−30	1	14.37	5	.....	.....	.....	.....	15.02	14.37	3, 5	0.65	0.65	5	0
127	14.35	6	.....	.....	− 4	2	13.31	11	.....	.....	.....	.....	14.34	13.31	8,11	1.03	0.89	12	+14
130	15.26	2	.....	.....	−21	1	14.21	9	.....	.....	.....	.....	15.19	14.21	3, 9	0.98	0.60	3	+38
134	14.43	6	.....	.....	−17	1	13.54	10	.....	.....	.....	.....	14.41	13.54	7,10	0.87	.....	.....	.....
151	13.72	29	.....	.....	− 2	3	12.58	11	.....	.....	+18	9	13.72	12.66	32,20	1.06	1.14	13	− 8
152	14.56	21	.....	.....	+12	3	13.76	12	.....	.....	− 3	1	14.58	13.76	24,13	0.82	0.68	12	+14
158	15.10	16	.....	.....	0	1	14.57	5	.....	.....	.....	.....	15.10	14.57	17, 5	0.53	0.64	3	−11
176	14.33	36	.....	.....	+14	5	13.81	12	.....	.....	−13	4	14.35	13.78	41,16	0.57	0.59	6	− 2
226	14.93	46	.....	.....	− 8	25	14.23	7	.....	.....	−21	4	14.90	14.15	71,11	0.75	0.82	8	− 7
256	14.67	50	.....	.....	− 4	28	13.86	11	.....	.....	− 4	8	14.66	13.84	78,19	0.82	0.92	10	−10
277	15.30	62	.....	.....	+ 6	49	14.61	7	.....	.....	−15	4	15.33	14.56	111,11	0.77	0.72	9	+ 5
311	15.00	53	.....	.....	+ 4	42	14.36	7	.....	.....	− 5	6	15.02	14.34	95,13	0.68	0.56	10	+12
319	14.44	40	.....	.....	+ 9	14	13.55	3	.....	.....	+ 7	25	14.46	13.61	54,28	0.85	0.73	10	+12
329	14.32	38	.....	.....	+10	14	13.34	3	.....	.....	+19	27	14.35	13.51	52,30	0.84	0.76	11	+ 8
334	15.20	70	.....	.....	− 1	51	14.42	7	.....	.....	+ 3	14	15.20	14.44	121,21	0.76	0.86	9	−10
338	14.72	28	.....	.....	+ 8	5	13.79	3	.....	.....	+ 6	10	14.73	13.84	33,13	0.89	0.82	8	+ 7
341	15.16	26	.....	.....	− 6	8	13.92	3	.....	.....	+12	15	15.15	14.02	34,18	1.13	1.06	8	+ 7
345	13.79	58	.....	.....	− 1	36	12.01	11	.....	.....	+ 5	35	13.79	12.05	94,46	1.74	1.50	29	+24

350.....	11.01	46	.....	.....	+ 3	26	10.64	12	.....	.....	+14	11	11.02	10.71	72,23	+0.31	+0.24	51	+ 7
358.....	15.22	26	.....	.....	+ 9	9	14.11	2	.....	.....	+47	2	15.24	14.34	35, 4	0.90	0.81	7	+ 9
362.....	13.13	86	.....	.....	−12	67	12.31	12	.....	.....	− 5	61	13.08	12.27	153,73	0.81	0.82	47	− 1
378.....	14.14	54	.....	.....	− 2	64	12.74	5	.....	.....	+ 8	54	14.13	12.81	118,59	1.32	1.33	32	− 1
389.....	13.57	43	.....	.....	+ 8	48	12.83	8	.....	.....	+ 8	43	13.61	12.90	91,51	0.71	0.62	17	+ 9
391.....	14.63	38	.....	.....	− 2	46	13.85	6	.....	.....	− 2	32	14.62	13.83	84,38	0.79	0.75	11	+ 4
401.....	14.53	36	.....	.....	− 8	25	13.61	3	.....	.....	+10	33	14.50	13.70	61,36	0.80	0.77	10	+ 3
403.....	14.51	41	.....	.....	− 4	27	13.75	3	.....	.....	+ 9	34	14.49	13.83	68,37	0.66	0.72	9	− 6
424.....	14.51	29	.....	.....	+ 2	20	13.64	3	.....	.....	+15	26	14.52	13.77	49,29	0.75	0.58	10	+17
426.....	13.83	28	.....	.....	+ 7	12	13.33	3	.....	.....	+ 6	14	13.85	13.38	40,17	0.47	0.55	10	− 8
427.....	13.27	33	.....	.....	− 1	23	12.07	5	.....	.....	− 2	41	13.27	12.05	56,46	1.22	1.05	24	+17
439.....	(15.29)	1	.....	.....	−51	21	13.49	3	.....	.....	.....	.....	14.78	13.49	21, 3	1.29	1.39	10	−10

be accepted as substantially correct, it follows that the means may be regarded as corrections to the original photo-visual scale of *Contribution* No. 97.

Table VII shows the results found in this manner, both provisionally from series *a* and finally from the data in Tables VI and VIII supplemented by the faint stars appearing in series *f* alone. The means depend upon weights approximately proportional to the number of values of log E, unless C_m is much less reliable than usual. Technically, this is not the best method of procedure, but practically the final differences are little affected by the system of weighting or the grouping. It is noteworthy that the provisional corrections to the photo-visual scale given in the second column of Table VII, as far as they go, are in close agreement with those in the fifth column based upon much more extensive data.

With the exception of differences of −0.07 and −0.09 at magnitudes 8.0 and 9.5, the corrections are insignificant, and show that within very small limits of error the photographic and photo-visual scales of *Contribution* No. 97 are mutually consistent as far as the seventeenth photo-visual magnitude (about 18 photographic). The two discordant points indicate a small local systematic error, but its source cannot certainly be located.

The investigation with the 10-inch refractor[1] makes a negative correction to the photo-visual scale rather improbable. At the same time, the exposure-ratio observations are not very uniformly distributed, and some of them may be systematically affected by neglected plate constants. For the present it seems best to let the magnitude scales stand as they are, and to regard the results from exposure ratios as a check which establishes the mutual consistency of the scales within a few hundredths of a magnitude.

4. COLLECTION OF RESULTS. ADOPTED MAGNITUDES

The various results on the magnitudes and colors of stars near the Pole can now be brought together and combined into means. These results include: (*a*) the original determination in Table IX of *Contribution* No. 97, made with the 60-inch reflector; (*b*) the

[1] *Loc. cit.*

determination with the 10-inch Cooke refractor, in Tables III and VI of *Contribution* No. 234; (*c*) the revision for the reduction of accidental error, discussed in section 1 above; and (*d*) the values of the color-indices found from exposure ratios.

Assuming the photographic scale of (*a*) to be correct between magnitudes 10 and 15, (*b*) confirms the photographic scale of (*c*) between magnitudes 6 and 14, and the photo-visual scale between 6 and 12; (*d*) shows that the two scales are mutually consistent to the seventeenth photo-visual magnitude, with a maximum difference of 0.09 mag. at the ninth magnitude.

The essential details of the combination of these results are in Table VIII. The stars of the Polar Sequence, arranged in order of photographic magnitude, are given as far as Nos. 35 and 16*s*, where the revisions cease. The latter part of the table includes 30 additional stars in order of the *H.A.* **48** number.

The data in the first three and the eighth and ninth columns are copied directly from Table IX, *Contribution* No. 97. The results from (*b*) are in the fourth and fifth, and tenth and eleventh columns, and have been transcribed directly from the last two columns of Tables III and VI, respectively, of *Contribution* No. 234. Those from (*c*) are in the sixth and seventh, and twelfth and thirteenth columns, and have been taken from Table I above. The algebraic signs are such that when the differences are added algebraically to the magnitudes of *Contribution* No. 97 they give the results of the various revisions.

The final magnitudes are in the fourteenth and fifteenth columns. These are the means of the results in columns two to seven, and eight to thirteen, respectively, weighted according to numbers of values involved in each, excepting that the polar-comparison values for star 22 have received half the usual weight. Resulting color-indices, C_m, and the total numbers of observations are in the sixteenth and seventeenth columns. The results from exposure ratios in the eighteenth and nineteenth columns have already been discussed.

With the exception of the stars in Table IX of *Contribution* No. 97 not included in the revision, which stand as they are,

Table VIII above summarizes all of the work done at Mount Wilson on the magnitudes and colors of stars near the Pole. The table itself gives little indication of the labor which lies behind it, the great burden of which in the measurement and reduction of the plates has fallen upon Miss Richmond and Miss Joyner of the Computing Division. I am deeply indebted to them for their part in an investigation which could not have been completed without generous assistance.

MOUNT WILSON OBSERVATORY
March 1922

ON THE SPECTRUM OF NEUTRAL HELIUM[1]

By LUDWIK SILBERSTEIN

ABSTRACT

Quantum theory of spectrum lines of neutral helium.—No attempt has hitherto been made to account for any of these lines because, since the neutral atom has two electrons besides the nucleus, the theoretical explanation is generally supposed to depend on the solution of the problem of three bodies. But while we do not possess a general solution of this problem, particular solutions are known and it is of interest to see what results they give. For the case of permanent collinearity of the three bodies, we get a spectral formula of the Balmer type, $\nu=(49N/8)\ (1/n^2-1/m^2)$, but this represents none of the observed lines. If, however, we assume that the mutual perturbation of the electrons is negligible, the formula comes out $\nu=4N[1/n_1^2-1/m_1^2+1/n_2^2-1/m_2^2]$; and if the proper integral values are taken, up to 9 for the n's and up to 32 for the m's, this formula gives 84 out of the 111 observed lines, in general within a fraction of an angstrom. Although the resulting classification does not correspond to the usual series arrangement, the success of the formula suggests that the *force between intra-atomic electrons* may be much less than that given by the Coulomb law.

It is but too well known that, up to the present, the quantum theory of spectrum emission has proved utterly unable to deal satisfactorily with any spectra but those of atomic hydrogen and, what essentially is the same thing, of ionized helium. It being generally admitted that the neutral helium atom consists of a nucleus and two electrons, the notoriously prohibitive nature of the general problem of three bodies was invoked by the leading spectroscopists as a sufficient excuse for not as much as attempting to account theoretically for a single one out of the hundred and odd lines observed in the emission spectrum of that element.[2] The more so, of course, in the case of atoms containing three or more electrons. But though it be true that we are as far as ever from possessing a general solution of that famous problem (not Sundman's, of course) in anything like a practicable form for the purpose in hand, yet some particular solutions are known, simple enough to be treated at once by the quantitizing method of modern spectroscopy, and

[1] *Communication No. 156* from the Research Laboratory of the Eastman Kodak Company.

[2] Thus, e.g., Sommerfeld, passing from hydrogen to neutral helium, appeals to Goethe's Faust ("Hier stock ich schon, wer hilft mir weiter fort?") and concludes the section by consoling himself with the fact that "so also were the astronomers, in spite of centuries of attempts, unable to solve satisfactorily the three-body problem." *Atombau und Spektrallinien.*

it has seemed worth while to try the possibilities in this direction and thus, perhaps, to break the spell which has hitherto paralyzed all desire of attacking the non-hydrogenic spectra.

At first the writer thought of applying the well-known particular solution of the problem of three bodies given by Lagrange as long ago as 1772. But the corresponding spectrum formula, which is easily deduced, proved impotent to cover as much as a single observed line of neutral helium. In fact, what of Lagrange's solution remains for the purpose in hand is the case of permanent *collinearity* of the three bodies, the two electrons describing (apart from relativistic refinements) equal and coplanar ellipses around the nucleus as one of the foci, with overlapping major axes and the other foci situated on opposite sides of the nucleus.[1] Each electron is then easily seen to move as if it had only the charge $-\frac{7}{8}e$ (instead of $-e$) and as if the other electron were absent. The total energy of the system is found at once to be, with a as major semi-axis,

$$E=-\frac{7e^2}{4a}=-\frac{49}{8}\frac{2\pi^2 m_0 e^4}{h^2(n'+n'')^2},$$

where n' and n'' are the azimuthal and the radial quantum numbers defining any stationary state, and the remaining symbols have their usual meaning. Writing $n=n'+n''$ for the total quantum number and introducing the Rydberg constant, $N=2\pi^2 m_0 e^4/ch^3$, we have

$$E=-\frac{49}{32}\cdot\frac{4Nhc}{n^2}.$$

Bohr's older model of the passive (non-emitting) He atom corresponds to the case $n=n'=1(n''=0)$, when both electrons describe the same circle; it gives $E=-\frac{4}{8}^9 Nhc$, whence also the familiar (and somewhat too large) ionization potential, which need not detain us here. By the last energy formula, with n for the final and m for the initial quantum number, we have the spectrum formula

$$\nu=\frac{49}{8}N\left(\frac{1}{n^2}-\frac{1}{m^2}\right),$$

[1] This is but a special case of any number of equal ellipses distributed symmetrically around the nucleus, as used by Landé and others for a coarse representation of the higher atoms for X-ray purposes.

which is of the simple Balmer type, each frequency ν being merely that of ionized helium magnified $\frac{4}{3}\frac{9}{2}$ times. Since in the present case there is no wobbling of the nucleus, we have to take, instead of the usual He value, $N=N_\infty$, i.e., about 109737 cm^{-1}. Ultimately, therefore, the spectrum formula corresponding to the class of *collinear* solutions becomes

$$\nu=672139\left(\frac{1}{n^2}-\frac{1}{m^2}\right).$$

Now, if one may judge from a week's personal experience spent over a multitude of trials, this formula does not cover any of the observed helium lines, at least as tabulated in Fowler's last *Report*.[1] The nearest "hit" was $n=4$, $m=7$, giving $\lambda_{air}=3535$, but even this was full two angstroms below the nearest observed line which is at about $\lambda=3537$. Of course, one felt from the outset that by such an extremely narrow class of solutions, as are the collinear ones, one could not reasonably expect to cover more than a very small part of all the observed lines, perhaps one or two only. But the formula refused to yield a single one.

In fine, the collinear solutions, though rigorous, do not represent any of the observed lines of neutral helium.[2]

Such being the case, the idea occurred to me that one might perhaps be more successful with some broader, though only approximate, solutions of the three-body problem, and as such the most simple, nay trivial, class of motions suggested itself, namely those in which *the mutual perturbation of the two electrons is practically nil or negligible*. At first the idea seemed almost repugnant, through its very banality, but on trial it proved surprisingly useful, as will appear hereafter.

As a matter of fact, we have no good evidence that the electrons, especially as trabants of the nucleus within the atom, do act upon each other at all, and some bold modern physicist, encouraged by the recognized prohibition to radiate, might deny the electrons

[1] A. Fowler, *Report on Series in Line Spectra*, Phys. Soc. London, 1922, pp. 93–94. I have taken account also of some special papers quoted by Fowler.

[2] This result throws even, in the writer's opinion, a flood of doubt upon the admissibility of Bohr's older model of the passive He atom, which belongs to the same class of solutions, as already mentioned.

the right to interact while they are busy obeying the orders of the central body driving them around on stationary orbits. But it is safer to put the restriction as above, i.e., as an approximate assumption. Whether at all and how often such cases of practically negligible mutual perturbation actually occur within the atom of helium (and similarly in other atoms), cannot be decided beforehand, but only by experience. If the corresponding spectrum formula works and covers some of the observed lines, it can be reasonably inferred that such comparative independence of the electrons does occur, statistically, in a good number of individual atoms. If no lines are covered, the conclusion will be that such a state of things does not occur at all or only very sporadically.

The spectrum formula corresponding to this class of motions can be written down at once. If n_1, n_2 (and similarly m_1, m_2) be the total quantum numbers of the first and the second electrons, the total energy of the system in any stationary state is simply the sum of

$$E_1=-\frac{4Nhc}{n_1^2} \quad \text{and} \quad E_2=-\frac{4Nhc}{n_2^2}.$$

The required formula is, therefore,

$$\nu=4N\left[\frac{1}{n_1^2}+\frac{1}{n_2^2}-\frac{1}{m_1^2}-\frac{1}{m_2^2}\right], \tag{1}$$

where n_1, etc., are four independent integers, fixing the final and the initial state, or, introducing a conveniently short symbol for the bracketed expression,

$$\nu=4N\left(\frac{m_1,\ m_2}{n_1,\ n_2}\right). \tag{1a}$$

We shall also speak of the spectrum line having this frequency as "the line m_1, m_2/n_1, n_2." This formula amounts, obviously, to putting

$$\nu=\nu_1+\nu_2, \tag{2}$$

where ν_1,ν_2 are, apart from a possible slight difference of N, the frequencies of any two lines of ionized helium, observed or only theoretical. This would then be a new kind of the combination

principle: the sum of frequencies belonging to one atomic system (ionized helium) giving the frequency for *another system* (neutral helium).

As to the value of N, it is certainly not rigorous to take that found experimentally for ionized helium or that (but slightly different) used by empirical spectroscopists for neutral helium. To be accurate, somewhat different values of N should be used for different lines, according to the nature and the mutual configuration of the two pairs of orbits (influencing the amount of wobbling of the nucleus), and these being unknown, one would have to adopt some statistically computed compromise value. But such niceties may be postponed to a future opportunity, and since for the present it will be enough to write all wave-lengths to four or five figures, we shall, without much ado, adopt in formula (1) the value $N = 109723.2$ used for He^+ in Fowler's *Report*, the more so as in using the form (2) it will be convenient to take directly ν_1 and ν_2 as given by Fowler (p. 95) for the observed frequencies of ionized helium. Thus we shall have ultimately, in cm.$^{-1}$,

$$\nu = 438893\left(\frac{m_1, m_2}{n_1, n_2}\right). \qquad (3)$$

In what follows λ_0 will be written for the wave-length in vacuo, the reciprocal of ν, and λ for λ_{air}. All observed values will be quoted from Fowler's *Report*, where the wave-lengths are in I.A. For the present no other material, scattered in special papers (and doubtful as to the origin of the lines), will be taken into account.

Passing now to try out the formula (3) or (2), let us notice that any line of the type $m_1, m_2/n, n$ will be an additive combination of two members of one and the same series of ionized helium. But we shall have also interesting cases of $n_1 \mp n_2$, when the addends belong to different He^+ series, not necessarily both observed, and yet give rise to an observed He line.

Lines of the type $m_1, m_2/2, 2$, i.e., derivable from the Lyman series of He^+, need not detain us here, since they all fall between 698 and 498, a region thus far not sufficiently explored.

For the same reason, though to a lesser extent, we may omit for the present the He lines derivable from Fowler's or the principal

series of He^+, these lines, of the type $m_1, m_2/3, 3$, stretching all beyond 1903 A into the ultra-violet.

Let us, therefore, pass at once to lines of the type

$$\frac{m_1, m_2}{4, 4}$$

i.e., derivable from the Pickering series. We can now use directly the formula (2) adding up pairs of Pickering frequencies. Thus, and writing (5), (6), etc., for the values of m_1 or m_2, we find the following interesting results:

$$\frac{5, 14}{4, 4} \qquad \begin{array}{l}(5)\quad 9875.1\\(14)\ 25191.8\end{array}$$

$$\nu = 35{,}067;\ \therefore \lambda_0 = 2851.7 - 0.8$$

$$\lambda = 2850.9;$$

the nearest line observed (Fowler, p. 94) has $\lambda = 2851$, and is tabulated among the combination lines of neutral helium;

$$\frac{5, 20}{4, 4} \qquad \begin{array}{l}(5)\quad 9875.1\\(20)\ 26333.6\end{array}$$

$$\nu = 36{,}209;\ \lambda = 2761.8 - 0.8 = 2761.0,$$

nearest line observed for He (*loc. cit.*) $\lambda = 2761$;

$$\frac{5, 25}{4, 4} \qquad \begin{array}{l}(5)\quad 9875.1\\(25)\ 26728.7\end{array}$$

$$\nu = 36{,}604;\ \lambda = 2732.0 - 0.8 = 2731.2,$$

nearest line observed for He (*loc. cit.*) $\lambda = 2732$.

These three agreements are remarkably close. The next but one theoretical line is

$$\nu = 4N\left(\frac{5, 18}{3, 8}\right) = 36{,}713.3;\ \lambda = 2723.8 - 0.8 = 2723.0,$$

the nearest He line (*loc. cit.*), observed by Merton, being λ 2722.

Notice that we could take also $m_1 = m_2$, which would mean simply doubling the frequency of a Pickering-series member, i.e.

taking $\nu_1 = \nu_2$. But none of these theoretical lines m, $m/4$, 4 fits an observed He line, even the nearest hit (5, 5/4, 4) with $\nu = 19{,}750$ being too distant from the observed He line $\nu = 19{,}931$.

We will thus content ourselves with the three or four good results derived from the Pickering series alone, and pass on to consider a few lines of the type

$$\frac{m_1, m_2}{5, 4},$$

i.e., combinations of the members of the Pickering series with those of a purely theoretical He^+ series $\nu_i = 4N(1/5^2 - 1/m_i^2)$, never observed. Notwithstanding the latter circumstance, neutral helium shows at least three lines agreeing admirably with the theoretical lines of this type. In fact, we have, always by formula (3),

$$\left(\frac{6, 9}{5, 4}\right) = 0.0623765,\ \nu = 27{,}377,\ \lambda = 3652.8 - 1.0 = 3651.8,$$

the nearest observed He line being 3652.0, tabulated in the sharp doublet series. Next,

$$\left(\frac{6, 17}{5, 4}\right) = 0.0712620,\ \nu = 31{,}276,\ \lambda = 3197.3 - 0.9 = 3196.4,$$

the nearest observed He line being 3196.7 (principal, $1S - mP$, *loc. cit.*), and finally,

$$\left(\frac{6, 19}{5, 4}\right) = 0.0719521,\ \nu = 31{,}579,\ \lambda = 3166.6 - 0.9 = 3165.7,$$

which covers the observed He line 3166 (combin., *loc. cit.*). [See also the combination (4.15/3.6) in the table below.]

To close this preliminary report on well-fitting lines it may be well to quote at least two examples in which both of the addends (ν_1 and ν_2) are purely theoretical. The first of these is

$$\left(\frac{6, 28}{5, 5}\right) = 0.0509467,\ \nu = 22{,}360,\ \lambda = 4472.25 - 1.25 = 4471.00,$$

with the nearest observed He line at $\lambda = 4471.48$ (diffuse doublet series, *loc. cit.*, p. 93), and the second, which has seemed interesting

as it concerns an infra-red line,

$$\left(\frac{10,\ 15}{7,\ 14}\right)=0.0110658, \quad \nu=4856.7,$$

which agrees fairly well with the observed He line (sharp, $1P-mS$) at $\nu=4857.3$.

Whether these nine or ten lines of neutral helium can also be covered by some other pairs of initial and final states of the system, in which the interaction of the electrons *is* taken into account, must for the present remain an open question. For no such solutions are known or likely to be forthcoming soon. In the meantime one would be justified to conclude from these results that there is a good deal of mutual independence of the two electrons within the radiating helium atom, which is particulary surprising in the case of such small orbits (of both) as those corresponding to the quantum number 4. A quantitative estimate of the perturbations to be expected on the ordinary repulsion law between the electrons, together with possible conclusions concerning those theoretical lines that do deviate from the observed ones by a few angstroms, will occupy our attention in a future publication.

In much the same way one could try to cover some lines of lithium (without awaiting, of course, an amputation of two of its three limbs or electrons) by three pairs of terms, i.e., by the formula $\nu=9N(m_1, m_2, m_3/n_1, n_2, n_3)$, and some spectrum lines of the higher atoms by four, five and more pairs of terms. But the larger the number of electrons, the less likely are the states of their comparative independence. Moreover, the larger the number of independent term pairs, the less convincing will be even a thorough fitting, so that it does not seem worth while to push the procedure much beyond lithium. At any rate the case of neutral helium has seemed the most interesting one, and an analogous treatment of lithium is now in progress.

ROCHESTER, N.Y.
July 4, 1922

ADDENDUM

Since the foregoing was sent to the Editors of the *Astrophysical Journal*, I have subjected all the observed spectrum lines of neutral helium to a kind of arithmetical sieve which enabled me to sweep the field with comparative ease. The result was, much to my own

surprise, that practically the whole spectrum of helium is covered by the proposed formula

$$\nu = 4N\left(\frac{m_1 \cdot m_2}{n_1 \cdot n_2}\right),$$

in the majority of cases within a fraction of an angstrom, the same value N, namely 109723.2,[1] being used throughout.

The numerical results, ordered for the present according to the total quantum numbers defining *the final state* $(n_1 \cdot n_2)$, are given in the following tables, with the calculated and the observed wave-number ν in the second, and the excess $\delta\lambda$ of the calculated over the observed wave-length, in angstroms, in the third column. The last column gives the empirical qualifications of the observed lines, with the abbreviations

P = principal, S = sharp, D = diffuse, F = fundamental, d = doublet system, s = singlet system, C = "combination,"

and with the intensities, where available, expressed by the bracketed numbers. The whole observed material is taken from Fowler's *Report*, already quoted, with the only addition of ν 21866 (λ 4572), observed by Stark in a strong electric field,[2] thus making a total of 111 observed lines. The initial and final orbit numbers are given in the first column. Spectrum lines of the type (m/n) can manifestly be interpreted as corresponding either to a jump of one electron from m to n, while the other electron remains on any stationary orbit, or else as corresponding to a jump of one electron from m to any integer x, and of the other electron from x to n,

$$\left(\frac{m}{n}\right) = \left(\frac{m \cdot x}{n \cdot x}\right) = \left(\frac{m \cdot x}{x \cdot n}\right).$$

Such, for instance, is the line $(6/3)$, written $(6 \cdot n/3 \cdot n)$, and similarly for the remaining five lines of this type. They are interesting inasmuch as they might also be produced by ionized helium, as far as mere arithmetic goes. All other lines are of the irreducible type $m_1 \cdot m_2/n_1 \cdot n_2$. That the classification according to n_1, n_2 lacerates the usual empirical classification (principal, sharp, etc.) may be regrettable but need not detain us here. Moreover, now and then,

[1] This was fixed provisionally and may still be slightly retouched, but not until one comes to consider the relativistic refinements and the dynamics of the wobbling of the nucleus.

[2] *Annalen der Physik.*, **56**, 577, 1918.

TABLE I

$\frac{m_1 \cdot m_2}{n_1 \cdot n_2}$	ν calc. ν obs.	$\delta\lambda$		$\frac{m_1 \cdot m_2}{n_1 \cdot n_2}$	ν calc. ν obs.	$\delta\lambda$	
$\frac{13.n}{4.n}$	24,834 24,830	−0.6	D.d. (5)	$\frac{8.15}{4.5}$	36,178 36,171	−0.5	P.d. (2)
$\frac{18.n}{4.n}$	26,076 26,078	+0.4	D.s. (1)	$\frac{5.14}{4.6}$	19,827 $19,82_4$	−0.9	C.s.
$\frac{22.n}{4.n}$	26,524 26,526	+0.3	D.s. (1)	$\frac{5.16}{4.6}$	20,352 20,358	+1.5	C.s.
$\frac{26.n}{4.n}$	26,781.5 26,781.6	+0.01	S.d. (1)	$\frac{7.15}{4.6}$	28,715 28,716	+0.1	S.d.
$\frac{30.n}{4.n}$	26,943 26,938	−0.7	C.d.	$\frac{11.15}{4.6}$	34,045 $34,05_0$	+0.5	C.d.
$\frac{4.9}{3.5}$	$33,47_2$ $33,48_0$	+0.6	C.d.	$\frac{11.26}{4.6}$	35,346 35,337	−0.7	P.d. (4)
$\frac{4.18}{3.5}$	37,536 37,537.5	+0.1	P.d. (1)	$\frac{12.28}{4.6}$	36,015 $36,00_0$	−1.2	C.d.
$\frac{4.20}{3.5}$	37,794 37,798	+0.3	P.d.	$\frac{16.23}{4.6}$	37,078 37,079	+0.1	P.d. (1)
$\frac{4.15}{3.6}$	$31,57_6$ $31,57_6$	0.0	C.d.	$\frac{17.24}{4.6}$	37,341.6 37,342.7	+0.1	P.d. (1)
$\frac{4.14}{3.7}$	28,053 28,058	+0.7	S.d. (1)	$\frac{20.23}{4.6}$	37,695 37,683	−0.9	P.d.
$\frac{4.16}{3.7}$	28,577 28,574	−0.4	D.d. (1)	$\frac{6.10}{4.7}$	19,807 19,805	−0.5	S.s. (2)
$\frac{5.18}{3.8}$	36,713 36,711	−0.15	P.d. (1)	$\frac{8.19}{4.7}$	28,314 28,316.5	−0.3	D.d. (1)
$\frac{4.25}{3.8}$	27,490.6 27,489.3	−0.2	C.d.	$\frac{8.25}{4.7}$	28.827.9 $28,82_7$	0.0	C.s.
$\frac{4.19}{3.8}$	26,977.0 26,982.4	+0.7	D.d. (3)	$\frac{8.26}{4.7}$	28,881 28,883	+0.3	D.d. (1)
$\frac{5.14}{4.4}$	35,067 $35,06_4$	−0.2	C.d.	$\frac{8.27}{4.7}$	28,928 28,921	−0.8	D.d. (1)
$\frac{5.20}{4.4}$	36,208.7 $36,20_8$	0.0	C.d.	$\frac{8.28}{4.7}$	28,970 $28,97_1$	+0.8	C.s.
$\frac{5.25}{4.4}$	36,603 $36,59_2$	−0.8	C.d.	$\frac{9.14}{4.7}$	28,730 28,736	+0.6	D.d. (1)
$\frac{6.9}{4.5}$	27,377 27,374	−0.35	S.d. (1)	$\frac{9.26}{4.7}$	30,320 30,324	+0.4	P.s. (1)
$\frac{6.17}{4.5}$	31,276 31,274	−0.3	P.s. (1)	$\frac{15.30}{4.7}$	33,950 33,945	−0.4	P.d. (6)
				$\frac{28.32}{4.7}$	35,399 $35,40_1$	+0.1	C.d.

TABLE I—*Continued*

$\frac{m_1 \cdot m_2}{n_1 \cdot n_2}$	ν calc. ν obs.	$\delta\lambda$		$\frac{m_1 \cdot m_2}{n_1 \cdot n_2}$	ν calc. ν obs.	$\delta\lambda$	
$\frac{7.18}{4.8}$	23,977 23,980	+0.5	S.s. (1)	$\frac{6.16}{5.5}$	21,205.5 21,210.8	+1.2	S.d. (1), (3)
$\frac{9.12}{4.8}$	25,822.5 25,820.5	−0.3	D.s. (1)	$\frac{6.28}{5.5}$	22,360.1 22,357.7	−0.5	D.d. (6)
$\frac{9.18}{4.8}$	27,516 27,508	−0.9	D.d. (2)	$\frac{7.11}{5.5}$	22,527.4 22,518.6	+0.3	S.s. (1)
$\frac{9.20}{4.8}$	27,773 27,775	+0.2	S.d. (1)	$\frac{7.19}{5.5}$	24,938.6 24,935.2	−0.5	D.s. (1)
$\frac{9.21}{4.8}$	27,874.9 27,874.1	−0.1	C.d.	$\frac{7.21}{5.5}$	25,159 25,157	−0.3	C.s.
$\frac{9.27}{4.8}$	28,268 28,266	−0.3	S.d. (1)	$\frac{7.25}{5.5}$	25,452 25,460	+1.3	D.s. (1)
$\frac{10.11}{4.8}$	26,272 26,269	−0.6	D.s. (1)	$\frac{8.24}{5.6}$	22,128 22,125	−0.5	C.d.
$\frac{10.14}{4.8}$	27,661 27,665	+0.6	P.s. (3)	$\frac{10.20}{5.6}$	24,261.1 24,260.3	−0.16	S.d. (3)
$\frac{10.18}{4.8}$	28,545.0 28,544.4	−0.07	S.d. (1)	$\frac{11.22}{5.6}$	25,213 25,215	+0.3	P.s. (4)
$\frac{10.20}{4.8}$	28,802 28,795	−0.8	D.d. (1)	$\frac{12.27}{5.6}$	26,098 26,103	+0.85	C.d.
$\frac{11.14}{4.8}$	28,422.3 28,422.6	+0.04	S.d. (1)	$\frac{13.20}{5.6}$	26,053 26,047	−0.9	S.s. (1)
$\frac{12.13}{4.8}$	28,644 28,640	−0.5	S.d. (1)	$\frac{13.22}{5.6}$	26,244 26,247	+0.3	C.s.
$\frac{12.14}{4.8}$	29,001 28,997	−0.5	P.s. (2)	$\frac{13.24}{5.6}$	26,388.4 26,395.2	+1.0	S.s. (1)
$\frac{13.28}{4.8}$	31,132 31,129	−0.3	P.s. (1)	$\frac{14.20}{5.6}$	26,411 26,413	+0.3	D.s. (1)
$\frac{14.14}{4.8}$	29,810 29,802	−0.9	P.s. (1)	$\frac{14.21}{5.6}$	26,513.0 26,513.7	+0.1	S.s. (1)
$\frac{14.18}{4.8}$	30,695 30,687	−1.4	P.s. (1)	$\frac{15.15}{5.6}$	25,846.0 25,848.9	+0.4	S.d. (1), (2)
$\frac{16.20}{4.8}$	31,476.9 31,472.4	−0.45	P.s. (1)	$\frac{18.24}{5.6}$	27,630.6 27,632	+0.2	C.s.
$\frac{16.19}{4.8}$	31,358 31,361	+0.3	P.d. (8)	$\frac{20.25}{5.7}$	24,713 24,710	−0.6	C.d.
$\frac{11.24}{4.9}$	28,460.0 28,461.6	+0.20	D.d. (1)	$\frac{22.24}{5.7}$	24,843.9 24,843.96	+0.01	S.s. (1)

TABLE I—*Continued*

$m_1 \cdot m_2$ / $n_1 \cdot n_2$	ν calc. / ν obs.	$\delta\lambda$		$m_1 \cdot m_2$ / $n_1 \cdot n_2$	ν calc. / ν obs.	$\delta\lambda$	
14.14 / 5.8	19,935 / 19,932	−0.9	P.s. (6)	9.24 / 6.7	14,968 / 14,970	+0.9	D.s. (6)
16.23 / 5.8	21,869 / 21,86$_6$	−0.7	C.*	10.15 / 7.14	4856.7 / 4857.3	$\delta\nu$ −0.6	P.s. (20) also S.s.
22.25 / 5.8	22,804 / 22,802	−0.6	C.s.				
9.15 / 6.6	17,014.0 / 17,014.3	+0.1	D.d. (1), (10)	23.11 / 8.9	7819.2 / 7819.9	$\delta\nu$ −0.7	F.d. (1)

* Line λ 4572 observed by Stark in an electric field of 28500 volt/cm. Not contained in Fowler's *Report*.

a good portion of an empirical series, such as "P.s." at $n_1 \cdot n_2 = 4.8$, reasserts itself.

These are, thus far, 84 different observed lines of helium. Of the 27 observed lines thus left unaccounted for the infra-red ones could not, up to the present, be covered within less than $\delta\nu = \pm 10$, without exceeding $n = 9$ for the final orbits. Of the remaining lines I have thus far succeeded in covering a few more within less than 2 angstroms, while fourteen show a difference of 2 to 3, and one of about 5 angstroms. The corresponding numerical details may be omitted for the present.

Thus, to say the least, over eighty out of the 111 observed lines of He are well represented by the aforesaid formula whose structure is such as if *the two electrons did not influence each other at all*, each being acted upon only by the nucleus.

At first one might, perhaps, expect this formula to represent *only* such lines as correspond to a passage from an initial to a final state in both of which the two electrons are on the whole so distant from each other that their initial energy required by the Coulomb law,

$$\Phi = e^2/r_{12},$$

is negligibly small. If this were so, then the well-fitting cases would be simply as many approximate solutions of the three-bodies problem in its sanctioned form (Coulomb's law for each pair), and there would be nothing very surprising in those coincidences. But this is by no means the case. On the contrary,

the energy neglected, or rather deliberately disregarded by the writer, in all these cases would amount, in terms of ν, to several thousand units,[1] and the amounts neglected in the initial and the final states by no means compensate each other—and even if they did, the considerable force of interaction would in each case have modified thoroughly the orbits themselves, spoiling the fitting altogether. If there is at all any repulsion between the two electrons, then, to make it compatible with the eighty results obtained, it can scarcely amount to more than one-ten-thousandth of the Coulomb force. To say the least, we are entitled to strongly suspect the validity of anything like the usual repulsion law between the electrons within an atomic system. Evidence of the same kind is afforded by the spectrum of lithium for which the writer has recently found several remarkably close coincidences with

$$\nu = 9N\left(\frac{m_1 \cdot m_2 \cdot m_3}{n_1 \cdot n_2 \cdot n_3}\right),$$

to be described at an early opportunity. As a matter of fact, there is (apart from a few slightly defective computations of ionization potentials) no experimental evidence for the Coulomb repulsion between intra-atomic electrons, anyhow. On the other hand its denial gave us a huge majority of the spectrum lines of helium, not a single one of which has been accounted for by models based on such a repulsion nor, in fact, in any other way.

In conclusion it may be well, in anticipation of possible objections, to notice that the results obtained can by no means be explained away as so many chance coincidences. In fact, a laborious computation of *all* the theoretical lines of the aforesaid type with the restriction to $n \leq 8$ and $m \leq 20$, which need not be reproduced here, has shown that the probability of only twenty of our tabulated lines (in the interval $\nu = 22{,}000$ to $38{,}000$) hitting the observed lines within the actual $\delta\nu$ by mere chance is less than 10^{-11}. And if some, at least, of the remaining sixty items are taken into account, the odds against a chance hit will be found to be enormous.

August 2, 1922

[1] E.g., for states such as (4.4) it amounts as a simple computation will show, to more than 13,500 waves (units of ν).

THE VARIABLE DOUBLE STAR X OPHIUCHI[1]

By C. H. GINGRICH

ABSTRACT

Variable double star X Ophiuchi.—From direct measurements by Hussey and later by Van Biesbroeck it is known that the two components of this interesting system are about 0".22 apart and were in a nearly north and south line in 1921, and that the north component is variable. During 1921 van Maanen made 16 plates to determine the parallax, which came out 0".000±0".007. These plates have since been measured in declination to see whether any *displacement of the combined image* due to the variation in relative brightness of the components could be detected. With reference to exposures made near a maximum of the combined light, a mean displacement of about 0".15 to the south was found for a group of eight exposures made about midway between maximum and minimum. This result is in agreement with the previous conclusions mentioned above. On the plates measured there was no evidence of elongation of the image.

This interesting and rather exceptional object, whose position for 1900 is $\alpha = 18^h33^m24^s$, $\delta = +8°44'$, was noted as variable first by Espin in 1886. It is included in the list of variable stars in *Harvard Annals*, **55**, 22, as star 183308, and is there assigned a range in magnitude from 6.5 to 9.0 in a period of 335 days. Its spectral classification is Md. Subsequent study of the variability of this star by Mr. Leon Campbell of the Harvard College Observatory based upon mean light curves extending over the interval from the year 1904 to the year 1921 indicates that the range is from magnitude 6.7 to 8.8 in a period of 337 days; $M-m=$ 189 days, $m-M=148$ days.

Hussey in 1900 discovered that this object is a double star, the components being separated by 0".22 in position angle 195°.2. It appears as Hu 198 in the list of double stars published by him in the *Astronomical Journal*, **21**, 35 (No. 485), 1900.

Merrill, without realizing that the star was double, noticed that the spectrum of this object changed to such an extent from time to time as to suggest that there were two sources of light, and that at certain times the one predominated and at other times the other. Through correspondence with Van Biesbroeck, of the Yerkes Observatory, it was learned that in his program of remeasuring

[1] *Contributions from the Mount Wilson Observatory*, No. 238.

all the double stars discovered by Hussey his attention had been attracted to this object by its deep orange color, and that he was measuring the pair and observing the light variation for the purpose of determining which of the components was responsible for the variation of the pair or whether both components were variable. The mean results of six measures which he made in 1920 are

$$1920.668 \quad 171^{\circ}.1 \quad 0''.218.$$

Later Van Biesbroeck furnished a light curve of the pair from estimates of magnitude extending from July, 1920, to October, 1921, covering more than one period of the variable, and showing the date of a maximum to be 1921, May 5. He also deduced the conclusion from estimates of relative brightness of the two components that the "north preceding star is the variable one."

In May, 1921, van Maanen completed a series of plates for determining the relative parallax of this object, and from them deduced the following result:

$$\pi = 0''.000 \pm 0''.007, \ \mu_a = 0''.000 \pm 0''.020.$$

Van Maanen and Merrill suggested that, since the image is undoubtedly displaced because of the relative change in brightness of the components, it would be desirable to measure the plates of the parallax series in order to discover the amount and direction of this displacement and thereby obtain an independent determination of the distance between the two, and also an indication as to which was varying.

Since the plates were taken within the period during which Van Biesbroeck made his measures, which show the stars to be nearly in a north and south line, it was immediately clear that the displacement due to variation in magnitude would be almost entirely in declination. Consequently the plates were arranged for measurement with the stereocomparator in the same pairs that van Maanen had used for the parallax measures, but were measured in declination instead of in right ascension as he had done. As the plates had been taken for maximum parallax displacement in right ascension they were of necessity extremely

unfavorable to parallax determination in declination, but, as van Maanen had already shown the relative parallax to be very small, this feature was of no special importance for the present purpose.

It should perhaps be stated that on the plates measured there was no evidence of elongation of the image and that consequently the settings were made so as to bisect the image resulting from the combined light of the pair.

I used the light curve derived by Van Biesbroeck for obtaining the magnitude of the pair at the times when the several plates were taken and assumed the magnitude at minimum to be the magnitude of the invariable component, and by use of the formula

$$\log b_n = \log b_m - 4/10\,(n-m)$$

given in Young's *General Astronomy*, page 509, constructed the following table of values:

TABLE I

Plate No.	Date	Combined Mag.	Ratio of Light Intensity		Mag. Var.
			Var./Inv.	Inv./Comb.	
2675	1920, May 15	7.1	2.98	0.25	7.4
2699	25	7.0	3.37	.23	7.3
2802	Aug. 7	8.0	0.74	.58	8.9
2814	8	8.0	0.74	.58	8.9
2819	20	8.2	0.45	.69	9.5
2831	21	8.2	0.45	.69	9.5
2846	22	8.2	0.45	.69	9.5
3083	1921, Apr. 13	7.2	2.63	.28	7.6
3100	May 1	7.0	3.37	.23	7.3
3112	13	7.0	3.37	0.23	7.3

By using data determined by Campbell and by Van Biesbroeck it was found that the star was at maximum brightness on 1920, June 2, and 1921, May 5, and at minimum brightness on 1919, November 25; 1920, October 28; and 1921, September 30. The mean date of the first two plates, May 20, is only thirteen days before the maximum of June 2, and the mean date of the last three, April 29, is only seven days before the maximum of May 5.

The mean date of the remaining five plates, August 18, is seventy-one days before the minimum of October 28.

In the reductions of the measures the factors in the next to the last column in Table I were taken to represent the proportionate displacement of the image from the position of the variable star toward the position of the invariable one. It is probable that the displacement of the image of the combined light due to variation in one of the components does not take place rigorously according to the law implied in this method. However, in the absence of accurate knowledge at this point this assumption is the best which can be made as a working hypothesis.

At first that part of Van Biesbroeck's measured distance between the stars, namely 0″.218, indicated by the assumption just referred to, was applied to the measured quantities in a direction consistent with the conclusion that the north star is the variable one, and the residual quantities were used for determining parallax and proper motion in declination. However, because of the small parallax factors already referred to and because of the short interval of time involved, the probable errors of these determinations were so large as to make the values entirely unreliable and practically meaningless. Next an equation was formed for each pair of plates with the distance between the stars, as well as the parallax and proper motion, left as unknown. The same factors as before were used as coefficients of the unknown distance δ. A least-squares solution of these equations led to the following results:

$$\pi = -0''.088 \pm 0''.092; \quad \mu_\delta = -0''.037 \pm 0''.022; \quad \delta = 0''.348 \pm 0''.025.$$

Inasmuch as this computed distance differs considerably from the distance measured directly, corrections based upon the computed distance were applied to the plate measures as before and a new determination of the parallax and proper motion made. The results were

$$\pi = -0''.012 \pm 0''.102; \quad \mu_\delta = -0''.030 \pm 0''.015.$$

These values of π and μ_δ are at least possible, but the probable errors make it unsafe to attach very much weight to them, especially in the case of the parallax value.

It was then decided to remeasure the plates by comparing each available exposure with a given exposure chosen as the standard, in order to detect the displacement from this standard position. The exposure on the plate numbered 3100 was chosen as standard because the date of the exposure, 1921, May 1, so nearly coincided with the date of the observed maximum, 1921, May 5. In all, fifteen exposures were compared with this standard, and equations formed, coefficients for π, μ_δ and δ (distance) determined by the same method as before being used. The following results were obtained:

$$\pi = +0''.002 \pm 0''.063; \quad \mu_\delta = -0''.001 \pm 0''.012; \quad \delta = -0''.385 \pm 0''.024.$$

The value of the parallax agrees well with that obtained by van Maanen by measuring these plates in right ascension, as already stated. Even though the time interval is short the probable error of the determined proper motion is small, and hence the value of μ_δ may be relied upon with some degree of confidence.

As for the value of the distance, the sign shows the displacement to be to the south of the standard position, and, since the standard exposure was made at a period of maximum, a time when the variable star was dominant, this fact furnishes independent confirmation of the conclusion by Van Biesbroeck that the north star is the variable one. The numerical value requires further consideration. The displacements obtained from the several plates are shown in Table II.

TABLE II

Plate Nos.	Displacement	Plate Nos.	Displacement
2675_1–3100	$-0''.015$	2831_1–3100	$-0''.154$
2675_2–3100	− .035	2831_2–3100	− .194
2699_1–3100	+ .039	2846_1–3100	− .165
2699_2–3100	+ .003	2846_2–3100	− .167
2802_1–3100	− .126	3083_1–3100	− .035
2802_2–3100	− .115	3083_2–3100	− .002
2814_1–3100	− .168	3100 –3100	.000
2819_1–3100	−0 .190	3112 –3100	−0 .018

It has been shown that the relative parallax and proper motion of this object are very small, and their effect in displacing the

image is therefore negligible. Hence the measured displacements may be considered as due to the relative change in magnitude alone. The mean displacement of the first four and the last four exposures, all of which were made very near to a period of maximum brightness, is $-0''.008$; the mean displacement of the remaining eight exposures, which were made within an interval of fifteen days, although seventy-one days before minimum, is $-0''.160$. There is therefore a measured displacement of $-0''.152$ in the interval from near maximum to seventy-one days before minimum. As the time during which this displacement took place is only one-half the interval from maximum to minimum, one might be led to conclude that a displacement of twice this amount would result from plates taken at minimum. As the distance between the stars obviously cannot be less than the measured displacement, this conclusion would indicate a distance fairly accordant with the computed value obtained above. Table I, however, would contradict such a conclusion in that it shows that on the mean date, August 18, of the second group of plates the invariable contributes more than twice as much light as the variable, instead of only an equal amount, as would seem to be required if the resultant image is to have a position midway between the extremes.

It seems, therefore, impossible to reach a definite conclusion because of uncertainty in two respects: first, the exact magnitude of the invariable star upon which depend the relative amounts of light contributed by the two components at the times of the exposures and, second, the effect of variable intensities upon the position of the resulting image. Both of these difficulties would vanish in a case in which we knew that the variable was entirely ineffective at minimum and completely dominated the formation of the image at maximum, and if we had one series of plates at maximum and another at minimum. The latter condition is not fulfilled in this instance, and it is probable that the former is not. The assumption of 8.6 as the magnitude of the invariable leads, as is seen in Table I, to the conclusion that the invariable star is responsible for one-fourth of the total light at maximum and for all of the light at minimum. It does not seem likely that the variable becomes so faint at minimum as to be entirely without

any effect in the formation of the image at that time. Even though it may contribute a part of the light, it may, however, be such a small percentage as to be ineffective in displacing the measured center of gravity. In that case the computed value for the distance between the stars, 0.″385, would be too large. On the other hand the measured displacement, 0.″152, although apparently more than half the distance between the stars, is somewhat smaller than the distance. The true value for the distance would therefore lie between these two limits. A number of suppositions might be made which would lead from the foregoing conclusion to a value almost identical with the value obtained by Van Biesbroeck from direct measurement. We may therefore say, in conclusion, that this investigation shows without any doubt that the north star is the variable one, and if it does not confirm the value for the distance between the stars it most certainly does not contradict it. The writer expects to continue the study of this pair by securing plates at maximum and at minimum.

The suggestions by Mr. van Maanen and Mr. Merrill during this work were of great help and were thoroughly appreciated by the writer. Valuable assistance was rendered also by Mrs. Marsh, of the Computing Division, who checked the computations in the early part of the study.

Mount Wilson Observatory
May 1922

PARALLAXES OF STARS IN THE REGION OF B.D.+31°643[1]

By C. H. GINGRICH

ABSTRACT

Diffuse, irregular nebula, and stars around B.D.+31°643—A photograph made by Hubble with the 100-inch reflector is reproduced. From a series of plates taken by van Maanen with the 60-inch reflector, the *parallaxes and proper motions of 20 stars* with reference to six or seven among them which appear not to be related to the nebula, were determined. Five of these have *color-indices* from 0.74 to 0.95 in excess of normal and seem to be involved in the nebula. The mean of the parallaxes of these five stars is $0''.0095 \pm 0''.006$. This is therefore taken to be the *parallax of the nebula*, equivalent to a distance of 350 light years from the sun.

The star B.D.+31°643 is of special interest because it is apparently imbedded in the densest part of a diffuse, irregularly shaped nebula extending over the region surrounding this star. The photograph of this region published herewith is reproduced from a negative made by Mr. Hubble with the 100-inch Hooker telescope, November 9, 1920, and kindly loaned to me for this purpose. The exposure time of this negative was 180 minutes.

A series of plates of this same region, taken by Mr. van Maanen with the 60-inch reflector as a part of his regular parallax program, was begun in December, 1919, and finished in January, 1922. The stars which appear on the parallax plates are numbered on the photograph (Plate I), except the brightest star, whose image was too large for measurement. The one designated as π is the star B.D.+31°643. Mr. van Maanen gave me these plates to measure in order to find out whether, on the basis of the parallaxes, the stars could be classified in two groups, the one containing those stars which belong to the nebula and the other those which do not.

Another basis for a separation into such groups had already been suggested by Mr. Hubble, namely, a comparison of color-index with spectral type. He had noticed that in the case of certain of the stars the color-index was larger than would be

[1] *Contributions from the Mount Wilson Observatory*, No. 239.

accounted for by the spectral type, which led him to the conclusion that the excessive redness was due to the absorption of the blue light by surrounding nebulous matter.

In Table I the first column gives the numbers assigned to the stars on the parallax plates and corresponding to the numbering on Plate I; the second and third columns give, respectively, the distances of the various stars from the central star in right ascension and declination, in minutes of arc; the next five columns were

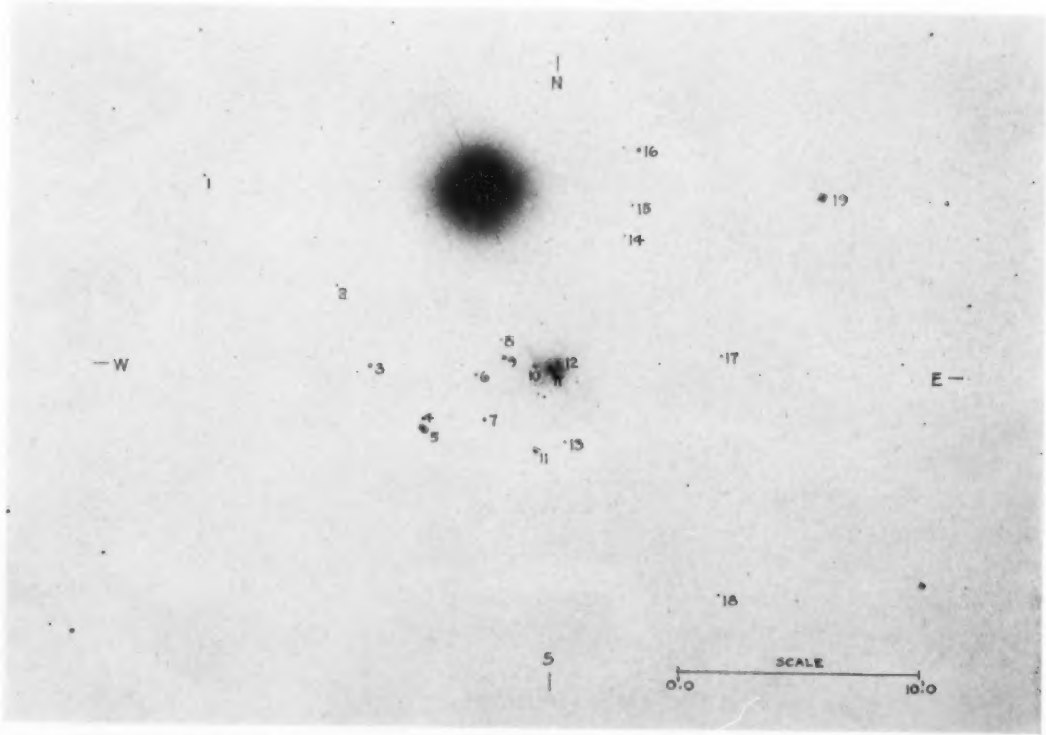
TABLE I

No.	x	y	Photo. Mag.	Spectrum	Observed Color-Index	Normal Color-Index	Color-Excess	π (Mean)	μ_α (Mean)
π	0′.0	0′.0	9.07	B6	+0.65	−0.14	0.79	− 8	− 14
1	−14.8	+7.7	.....	.....	.....	.....	.....	+ 67	− 56
2	− 9.0	+3.3	.....	.....	.....	.....	.....	− 17	− 34
3	− 7.6	0.0	11.81	.....	+ .59	.....	.....	+ 13	+ 25
4	− 5.5	−2.5	10.19	B8	+ .42	− .07	.49	+ 31	− 10
5	− 5.2	−2.7	10.31	B8	+ .48	− .07	.55	+ 28	− 15
6	− 3.2	−0.3	12.32	A2	+ .71	+ .07	.64	+ 8	+ 4
7	− 2.7	−2.2	12.08	F2	+ .65	+ .48	.17	+ 2	+ 10
8	− 2.2	+1.2	14.6±	.....	.....	.....	.....	− 14	− 5
9	− 2.1	+0.4	12.14	A1	+ .82	+ .03	.79	+ 28	+ 22
10	− 0.7	+0.1	12.32	B9	+ .71	− .03	.74	+ 5	− 5
11	− 0.5	−3.4	11.42	B9	+ .92	− .03	.95	− 5	− 9
12	+ 0.3	+0.3	10.80	B9	+ .80	− .03	.83	+ 32	+ 75
13	+ 0.7	−3.0	14.5±	.....	.....	.....	.....	− 44	− 85
14	+ 2.9	+5.8	14.8±	.....	.....	.....	.....	+ 24	+115
15	+ 3.2	+7.1	14.3±	.....	.....	.....	.....	+ 50	+ 75
16	+ 3.4	+9.5	11.99	A0	+ .96	.00	.96	+100	+210
17	+ 7.1	+0.7	14.0±	.....	.....	.....	.....	0	+ 15
18	+ 7.1	−9.2	14.22	.....	+1.02	.....	.....	− 36	−162
19	+11.1	+7.5	9.40	K0	+1.53	+1.48	0.05	+ 34	+ 29

supplied by Mr. Seares and Mr. Hubble from material now being prepared for publication by them; the last two columns give the final mean values of parallax and proper motion in right ascension, expressed in thousandths of a second of arc. These last quantities will be explained more fully later.

The plates were arranged in pairs and measured with the new stereocomparator of the Mount Wilson Observatory according to the method developed by van Maanen and fully described by him in *Mount Wilson Contribution* No. 111. For comparison stars those were chosen which, according to the color-index cri-

PLATE I



REGION OF B.D.+31°643 PHOTOGRAPHED BY HUBBLE WITH THE 100-INCH HOOKER REFLECTOR, NOVEMBER 9, 1920

terion, had no connection with the nebula. As was to be expected, this selection resulted in having as comparison stars those situated at some distance from the nebula and consequently located at considerable distances from the center of the plate. It was accordingly advisable to use quadratic terms in the reductions. However, as the exclusion of the stars which may be connected with the nebula left only seven stars available for that purpose, the six plate constants could not be determined with great accuracy. Two other solutions were therefore made, omitting the quadratic terms. The number of comparison stars in the first solution was seven and in the later ones six and four, omitting those so far from the center as to show unsymmetrical images on some of the plates.

The fact that the number of comparison stars available was small is probably not a matter of mere accident. On the parallax plates certain regions are easily noticeable in which no stars appear. It therefore seems likely that the nebula so conspicuous in certain parts of the field extends faintly over perhaps the entire region of the parallax plates with the result that the light of the fainter stars is absorbed or scattered by it. This conclusion is confirmed on the basis of the total number of stars on the parallax plates. According to *Groningen Publications* No. 27, the total number of stars would indicate that the faintest stars appearing are of magnitude 12.3 approximately, whereas as a rule stars as faint as magnitude 13.7 appear on the plates. In other words we should normally expect to find more than three times as many stars on plates at the galactic latitude of this field as are actually found on these.

Moreover, as will be seen in Table II, no definite classification of the stars into nebulous and non-nebulous is possible on the basis of the parallaxes found.

The values of parallax and proper motion in right ascension for each of the stars from each of the three reductions are shown in Table II. The first values resulted from a solution including quadratic terms and using stars 1, 2, 3, 7, 17, 18, 19 as comparison stars; the second, not including quadratic terms and employing stars 2, 3, 7, 17, 18, 19 as comparison stars; the third is like the

second except that only four comparison stars were used, 2, 3, 7, 17. The results are expressed in thousandths of a second of arc.

In order to determine the weights to be assigned to each of these sets of values the arithmetical means of the three were formed and the deviations from the means noted. These deviations indicated that weights 1, 1, 2 should be used. Adopting these weights the mean values of π and μ_a given in the last two columns of Table I were obtained. The mean probable error of all the parallaxes is $\pm 0''.013$ and of all the proper motions is $\pm 0''.019$. These values

TABLE II

SUMMARY OF π AND μ_a

	π_1	p.e.	μ_{a1}	p.e.	π_2	p.e.	μ_{a2}	p.e.	π_3	p.e.	μ_{a3}	p.e.
π	+ 36	8	− 2	12	− 22	7	+ 5	10	− 23	6	− 30	9
1	+ 11	3	+ 12	4	+ 57	31	−192	45	+100	19	− 21	28
2	− 24	5	− 30	8	− 25	11	− 75	16	− 10	3	− 16	4
3	− 7	3	+ 4	4	+ 24	7	+ 41	10	+ 18	5	+ 28	7
4	+ 26	9	− 18	14	+ 47	11	+ 38	16	+ 25	10	− 31	15
5	+ 25	2	− 20	3	+ 45	8	+ 37	12	+ 21	5	− 39	7
6	+ 40	9	+ 12	14	+ 2	11	+ 22	16	− 4	8	− 9	12
7	+ 26	5	+ 20	8	+ 7	9	+ 54	13	− 12	4	− 17	6
8	+ 33	11	+ 8	17	− 34	5	− 7	7	− 27	5	− 10	7
9	+ 70	11	+ 32	16	+ 13	8	+ 32	12	+ 14	6	+ 11	9
10	+ 50	21	+ 7	30	− 10	18	+ 12	26	− 10	20	− 19	29
11	+ 27	13	+ 17	19	+ 1	13	+ 51	19	− 24	8	− 52	12
12	+ 78	18	+ 88	26	+ 15	9	+ 91	13	+ 17	10	+ 61	15
13	− 10	11	− 58	16	− 40	10	− 31	15	− 64	12	−126	18
14	+107	33	+159	48	− 31	8	+ 51	12	+ 10	18	+125	26
15	+148	32	+138	46	− 16	8	− 13	12	+ 35	18	+ 88	26
16	+229	48	+319	71	+ 12	13	+ 75	19	+ 80	24	+224	35
17	− 1	1	+ 1	2	− 6	8	+ 48	12	+ 4	1	+ 5	1
18	− 5	1	− 5	2	− 6	7	− 51	10	− 66	45	−297	66
19	0	1	− 1	1	+ 5	4	− 18	6	+ 66	21	+ 68	31

are unusually large because stars 1, 16, 18, and 11 are more than 10′ from the center, the limit usually set for parallax determinations, and hence not well suited for measurement. Excluding these stars from the computation, we find the mean probable error of the remainder of the parallaxes to be $\pm 0''.009$, and of the proper motions $\pm 0''.010$.

A direct examination of the original negative from which the accompanying plate has been made would indicate that the stars numbered 1, 2, 3, 7, 14, 15, 17, 18, and 19 are not connected with the nebula; that those numbered 8, 13 are probably involved in it; and that those numbered π, 9, 10, 11, 12, 16 are apparently

enmeshed in it. There is no nebulosity visible about the stars 4, 5, 6, but their color-indices are such as to lead one to suspect the presence of nebulous matter.

Selecting from Table I those stars which to all appearances are imbedded in the nebula, omitting the one numbered 16 because of uncertainty due to its distance from the center of the plate and from the other stars of the group, we have for the five stars, π, 9, 10, 11, 12, a mean relative parallax of +0".0104. Two other stars, those numbered 3 and 6, have about the same parallax as the mean just found. The latter, as already stated, should probably be included among the five because of the evidence furnished by its color-index; as for the former, no evidence of connection with the nebula or with the group is indicated other than that of its parallax.

We are therefore led to the conclusion that the mean relative parallax of the five stars and hence the parallax of the nebula of which they almost certainly form a part is +0".0104. By applying to this the correction of −0".0009 derived by van Maanen[1] to reduce it to absolute parallax we find as the absolute parallax of the nebula +0".0095. This value of the parallax indicates a distance from the sun of about 350 light-years.

The very bright star in the photograph is *o* Persei. Two determinations of the parallax of this star have been made: one at the Sproul Observatory, +0".035; the other at the Allegheny Observatory, +0".003. The mean of these shows a value not greatly at variance with the parallax found for the nebula.

The double star 4 and 5 is listed as number 1832 in Burnham's *General Catalogue* and is there assigned a separation of 11".14 in position angle 128°.6. Measures extending from the year 1868 to the year 1903 show no change in either of these quantities.

The central star B.D.+31°643, designated on the plates used as π, is also a double star. It is numbered 1836 in Burnham's *General Catalogue* and consists of two stars of nearly the same brightness and only 0".45 apart in 1880. Measures from the year 1891 to the year 1893 indicate a slight progressive increase in the distance. On

[1] *Mt. Wilson Contr.*, No. 237.

the scale of our plates the two components combine to form a single symmetrical image.

This investigation would have been entirely impossible at this time had it not been for the series of plates taken by Mr. van Maanen, and in a sense therefore these results are due to him. Furthermore, his large experience in parallax work was heavily drawn upon throughout this work, and numerous constructive suggestions were cheerfully made by him.

The writer wishes also to express his hearty appreciation of the willing and generous co-operation of Mr. Seares and Mr. Hubble in the ways already indicated.

MOUNT WILSON OBSERVATORY
April 1922